HESSIAN STRUCTURES
VIA INVARIANT CALCULUS

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Dedicated to Bang-Yen Chen

Abstract

Consider a pseudo-Riemannian manifold (M, g) with the connection ∇ and a smooth function $f \colon M \to \mathbf{R}$ whose Hessian with respect to g is non-degenerate. Then we can define on M the associated pseudo-Riemannian Hessian metric $h = \nabla_g^2 f$, having the connection $\bar{\nabla}$. The goal of this paper is twofold. On the one hand we establish a link between the two connections ∇ and $\bar{\nabla}$ respectively. In this respect, we use invariant calculus. The proof of Theorem 2.1 is original. On the other hand we give the PDEs determined by the associativity of the deformation algebra when g has a particular form.

Of course, our theoretical presentation is not exhaustive, so the reader is encouraged to consult another recent works. The standard reference for Hessian structures is the monograph by Shima (2007). For additional background material on the subject of this paper, the authors recommend this treatise and the following research works Atanasiu (1973), Nicolescu and Udrişte (1976), Shima (1980), (1988-1989) and Shima and Yagi (1997).

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