COMPUTER PROGRAM FOR THE CUSUM CHART

Nicolae EFTIMIE[1]

Abstract: *The paper presents a computer program for the Cumulative Sum (or CUSUM) Control Chart. This type of chart represents an alternative to the Shewhart control chart for phase II process monitoring situations. The CUSUM chart takes into account the information given by the entire sequence of the points; therefore it can be used for detecting small changes in the process mean. The program was developed in MS Excel 2007, according to the Tabular CUSUM procedure and can be used in plant practice for monitoring the process mean. The paper presents also an observation about the way the Shewhart chart can detect the small changes in the process mean.*

Key words: *CUSUM chart, SPC, Computer Aided Quality.*

1. Introduction

The Cumulative Sum Control Chart takes into account the information given by the entire sequence of the points; therefore it can be used for detecting small changes in the process mean. This makes the CUSUM chart an alternative to the Shewhart control chart for phase II process monitoring situations [1].

There are two procedures of application for the CUSUM charts: the Tabular CUSUM procedure and the V-Mask CUSUM procedure. Of the two procedures, the tabular (or algorithmic) procedure is preferable [1].

When the process is in control, x_i has a normal distribution, with mean μ_0 and standard deviation σ [1]. In many situations, μ_0 represents the target value for the quality characteristic *x*.

In order to apply the Tabular CUSUM procedure, the statistics C_i^+ and C_i^- are calculated, which are called one-sided upper and lower cusums, respectively. They are computed by means of relations (1) and (2) [1]:

$$C_i^+ = \max\{0, x_i - (\mu_0 + K) + C_{i-1}^+\}, \quad (1)$$

$$C_i^- = \min\{0, x_i - (\mu_0 - K) + C_{i-1}^-\}, \quad (2)$$

where the starting values are $C_0^+ = C_0^- = 0$.

In equations (1) and (2), *K* is the reference value (or the allowance) and is chosen halfway between the target value μ_0 and the out-of-control value of the mean μ_1 that we are interested in detecting quickly [1]. If the shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$, then the value of *K* can be calculated by means of formula (3) [1]:

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2}. \quad (3)$$

The control limits of the chart are:

$$UCL = H \quad \text{and} \quad LCL = -H, \quad (4)$$

where $H = h\sigma$.

---

[1] Dept. of Manufacturing Engineering, *Transilvania* University of Braşov.

2. Objectives

The CUSUM chart represents an important tool that can be used for detecting the small changes in the process mean, in phase II of process monitoring situations.

The objective of the paper is to develop a computer program for the application of the CUSUM chart, in plant practice.

Using the input data, the program calculates the values of the statistics C_i^+ and C_i^- as well as the control limits. After that, the program detects the points that are situated beyond the control limits and calculates the periods when the shifts of the process probably occurred. In the last stage, the program plots the CUSUM chart.

The paper also presents an observation about the way the Shewhart chart can detect the small changes in the process mean.

3. Material and Methods

The computer program was developed in MS Excel 2007.

The program has an input data area that is used by the program's user for recording the values of μ_0, σ, δ, and *h* (Figure 1).

μ₀	σ	δ	h
25	0.1	1	5

Fig. 1. *The input data area*

At the beginning, the program calculates the values of *K* and *H* (Figure 2).

K	H
0.05	0.5

Fig. 2. *The values of K and H*

The measurements are recorded in the column labelled x_i (Figure 3).

The program first calculates the quantities x_i - $(\mu_0 + K)$, and then the values of the statistic C_i^+ (Figure 3).

In order to obtain the maximum value, the program uses the Excel function `MAX(number1, number2,…)` [2].

The quantity N^+ represents the number of consecutive periods when the statistic C_i^+ has been nonzero [1].

i	x_i	x_i - (μ_0+K)	C_i^+	N^+
1	24.94	-0.11	0.00	0
2	24.84	-0.21	0.00	0
3	25.07	0.02	0.02	1
28	25.09	0.04	0.49	7
29	25.08	0.03	0.52	8
30	24.99	-0.06	0.46	9

Fig. 3. *The calculation of C_i^+ and N^+*

In order to calculate the values of N^+ the computer program uses an Excel function `IF(logical_test,value_if_true, value_if_false)` [2] (Figure 3).

In the same manner, the program first calculates the quantities x_i - (μ_0-K), and then the values of the statistic C_i^- (Figure 4).

In order to obtain the minimum value, the program uses the Excel function `MIN(number1, number2, …)` [2].

The quantity N^- represents the number of consecutive periods when the statistic C_i^- has been nonzero [1].

In order to calculate the values of N^-, the Excel function `IF(logical_test, value_if_true,value_if_false)` [2] (Figure 4) was used.

Then, the program calculates the control limits, performs an analysis for finding the points beyond the control limits and calculates the periods when the process shifts probably occurred (Figure 5).

=IF(G9=0,0,H8+1)

$x_i - (\mu_0 - K)$	C_i^-	N^-
-0.01	0.01	1
-0.11	-0.12	2
0.12	0.00	0

=MIN(0,F9+G8)

0.14	0.00	0
0.13	0.00	0
0.04	0.00	0

Fig. 4. *The calculation of C_i^- and N^-*

In order to detect the points above the Upper Control Limit and the points below the Lower Control Limit, the program uses IF functions.

For the points that are beyond the control limits, the program calculates the periods when the process shifts probably occurred, using the procedure presented in [1].

The procedure is implemented by means of two IF functions.

In the presented example (Figure 5), the point 29 is above the Upper Control Limit.

As shown in Figure 3, it can be noticed that the value of N^+ for this point is equal to 8.

This means that the process was in control at period 29 – 8 = 21.

So, the process shift probably occurred between periods 21 and 22 (Figure 5).

UCL=H	$C_i^+ > H$	Process Shift	LCL= -H	$C_i^- < -H$	Process Shift
0.5	NO		-0.5	NO	
0.5	NO		-0.5	NO	

=IF(D8>I8,"YES","NO")

=IF(G8<L8,"YES","NO")

=IF(J8="YES","BETWEEN "&(A8-E8)&" AND "&(A8-E8+1),"")

0.5	NO		-0.5	NO	
0.5	YES	BETWEEN 21 AND 22	-0.5	NO	
0.5	NO		-0.5	NO	

Point 29 above UCL

=IF(M8="YES","BETWEEN "&(A8-H8)&" AND "&(A8-H8+1),"")

Fig. 5. *The control limits and the analysis of the chart*

4. Results and Discussions

Using the one-sided upper and lower cusums, and the control limits, the program plots the control chart (Figure 6).

The point 29 is situated above the Upper Control Limit. The special cause of variation that influenced the process must be found and removed from the process.

The paper also presents an observation derived from a large number of program's run (the program plots the Shewhart chart, too): in many situations, the Shewhart chart can detect the small changes in the process mean using the rule "Seven or more consecutive points situated on the same side of the Center Line".

Figure 7 presents the Shewhart chart, which corresponds to the same set of input data as that used for the CUSUM chart.
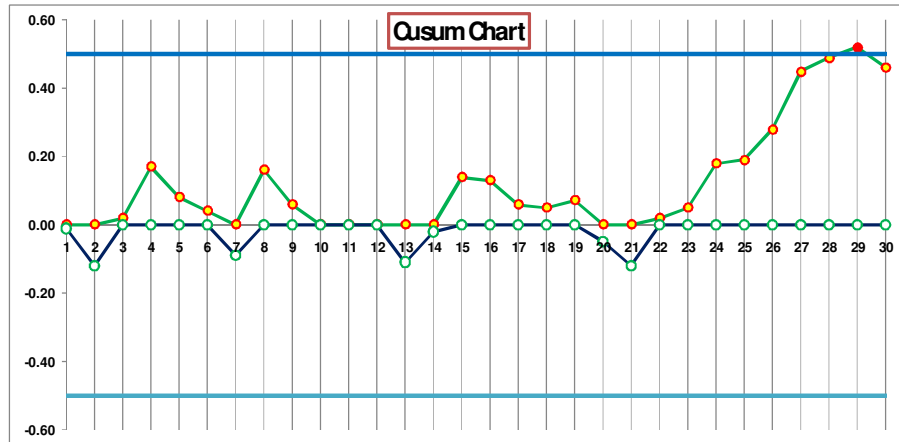
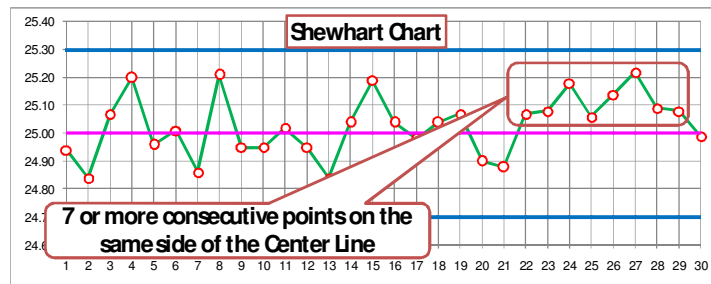Fig. 6. *The Cusum chart plotted by the program*

Fig. 7. *The Shewhart chart for the same set of input data*

The process shift is detected by the Shewhart chart: there are eight consecutive points situated above the Center Line.

5. Conclusions

The presented program can be used in plant practice for the application of the CUSUM chart.

The program will be used in phase II process monitoring situations, for detecting the small changes in the process mean.

The small changes in the process mean can be detected in many situations by the Shewhart chart, using the rule: "Seven or more consecutive points situated on the same side of the Center Line".

Acknowledgements

References

1. Montgomery, D.C.: *Introduction to Statistical Quality Control. Fifth Edition.* New York. John Wiley & Sons, Inc, 2005.
2. ∗∗∗ Microsoft Excel 2007 Help.