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STATICAL AND DYNAMICAL ANALYSIS OF COMPOSITE SANDWICH PLATES

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Abstract: *Sandwich plate with laminate facings and their equivalent was used for statical analysis. Plate with microwave zinc plated steel sheets and sandwich plate with laminate facings were used for dynamical analysis. Displacements, stresses and strains of sandwich plates were calculated and compared. The values of displacements and stresses were more accurate when the facings of laminate layers were used counter to their equivalent replacement. The facings of sandwich panel affect on its angular velocities and natural frequencies, while the lowest values were achieved by using of zinc plated steel sheet facings, what is related to their stiffness.*

Key words: *sandwich plates, displacements, stress, vibration.*

1. Introduction

A composite material can be defined as a heterogeneous mixture of two or more homogeneous phases, with their different physical properties, which have been bonded together. Properties of composite material are clearly distinct from the properties of its components. The most important aspect of composite materials in which the reinforcement are fibers is the anisotropy caused by the fiber orientation. It is necessary to give special attention to this fundamental characteristic of fibre reinforced composites and the possibility to influence the anisotropy by material design for a desired quality. The sandwich panels are one of the types of composite materials that are used in structures. They are usually composed of three layers.

The facings are made of the materials that have high strength (metals and fibre reinforced laminates) and are able to transfer axial forces and bending moments. A core is made of lightweight materials such as foam, alder wood etc. The material used in sandwich core must be resistant to compression and capable to transfer shear stresses [1], [2], [6].

2. First Order Shear Deformation Theory (FSDT)

FSDT is considering that the transverse normals do not remain perpendicular to the midsurface after deformation. This amounts to including transverse shear strains in the theory. The inextensibility of transverse normals requires that w not be a function of the thickness coordinate, z. It enables to use this theory for thicker plates or sandwiches, respectively [1], [4].

We are based on the assumption that the layers have certain stiffness in the transverse direction of the axis $3 \equiv z$. Transverse shear stresses are added to the state of plane stress for this reason. In the final form we get (L - local coordinate system):

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$$\boldsymbol{\sigma}_L^{(k)} = \mathbf{E}_L^{(k)}\boldsymbol{\varepsilon}_L^{(k)} \Leftrightarrow \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{pmatrix}^{(k)} = \begin{pmatrix} (E_{11})_L & (E_{12})_L & 0 & 0 & 0 \\ (E_{21})_L & (E_{22})_L & 0 & 0 & 0 \\ 0 & 0 & (E_{44})_L & 0 & 0 \\ 0 & 0 & 0 & (E_{55})_L & 0 \\ 0 & 0 & 0 & 0 & (E_{66})_L \end{pmatrix}^{(k)} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{pmatrix}^{(k)}. \quad (1)$$

Relationship between stress and strain for orthotropic material is characterized by the angle of rotation $\theta^{(k)}$ (Figure 1) of the material axis (1, 2, 3) to reference system (x, y, z).

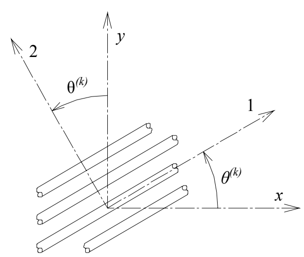

Fig. 1. *Rotation of material axis to reference system*

Internal forces are expressed by terms:

$$\mathbf{N} = \int_{-\left(\frac{1}{2}h^{(2)}+h^{(1)}\right)}^{-\frac{1}{2}h^{(2)}} \boldsymbol{\sigma} dz + \int_{\frac{1}{2}h^{(2)}}^{\frac{1}{2}h^{(2)}+h^{(3)}} \boldsymbol{\sigma} dz,$$

$$\mathbf{M} = \int_{-\left(\frac{1}{2}h^{(2)}+h^{(1)}\right)}^{-\frac{1}{2}h^{(2)}} \boldsymbol{\sigma} z dz + \int_{\frac{1}{2}h^{(2)}}^{\frac{1}{2}h^{(2)}+h^{(3)}} \boldsymbol{\sigma} z dz, \quad (2)$$

$$\mathbf{V} = \int_{-\frac{1}{2}h^{(2)}}^{\frac{1}{2}h^{(2)}} \tau dz.$$

The constitutive equations for a sandwich are in the form:

$$\begin{pmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}^s \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_\mathbf{m}^\mathbf{0} \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma} \end{pmatrix}, \quad (3)$$

with stiffness coefficients:

$$\begin{aligned} A_{ij} &= A_{ij}^{(1)} + A_{ij}^{(3)}, \\ B_{ij} &= \frac{1}{2} h^{(2)} \left(A_{ij}^{(3)} - A_{ij}^{(1)} \right), \\ C_{ij} &= C_{ij}^{(1)} + C_{ij}^{(3)}, \\ D_{ij} &= \frac{1}{2} h^{(2)} \left(C_{ij}^{(3)} - C_{ij}^{(1)} \right), \\ A_{ij}^s &= E_{ij}^s h^{(2)};\ i,j = 4, 5, \end{aligned} \quad (4)$$

where E_{ij}^s are the transverse shear moduli of the core.

2.1. Free Vibrations of Sandwich Plates

The equations to determine the natural frequencies of symmetric sandwich panel are used [8]:

$$D_{11}\frac{\partial^2\overline{\alpha}}{\partial x^2} + D_{66}\frac{\partial^2\overline{\alpha}}{\partial y^2} + (D_{12}+D_{66})\frac{\partial^2\overline{\beta}}{\partial x\partial y} - k^s A_{55}\left(\overline{\alpha} + \frac{\partial w}{\partial x}\right) - I\frac{\partial^2\overline{\alpha}}{\partial t^2} = 0, \quad (5)$$

$$(D_{12}+D_{66})\frac{\partial^2\overline{\alpha}}{\partial x\partial y} + D_{66}\frac{\partial^2\overline{\beta}}{\partial x^2} + D_{22}\frac{\partial^2\overline{\beta}}{\partial y^2} - k^s A_{44}\left(\overline{\beta} + \frac{\partial w}{\partial y}\right) - I\frac{\partial^2\overline{\beta}}{\partial t^2} = 0, \quad (6)$$

$$k^s A_{55}\left(\frac{\partial\overline{\alpha}}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+k^s A_{44}\left(\frac{\partial\overline{\beta}}{\partial y}+\frac{\partial^2 w}{\partial y^2}\right)-\rho_m h\frac{\partial^2 w}{\partial t^2}=0, \tag{7}$$

where k^s is the transverse shear deformation factor given by value 5/6:

$$\rho_m=\frac{1}{h}\sum_{k=1}^{N}\rho_k(z^{(k)}-z^{(k-1)}),$$
$$I=\frac{\rho_m h^3}{12}\frac{1}{3}\sum_{k=1}^{N}\rho_k(z^{(k)})^3-(z^{(k-1)})^3, \tag{8}$$

where ρ_k is the mass density of the k^{th} layer.

For the simply supported plate let:

$$w(x,y,t)=$$
$$=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}C'_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\mathrm{e}^{i\omega_{mn}t},$$

$$\overline{\alpha}(x,y,t)=$$
$$=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A'_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\mathrm{e}^{i\omega_{mn}t}, \tag{9}$$

$$\overline{\beta}(x,y,t)=$$
$$=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B'_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\mathrm{e}^{i\omega_{mn}t},$$

where: m, n - are integers only; a, b - are the panel dimensions in x, y axis direction respectively; ω_{mn} - is natural angular velocity.

Substituting term (9) into equations (5), (6) and (7) results in a set of homogeneous equations that are used to solve the natural frequencies of vibration:

$$\begin{pmatrix} L'_{11} & L_{12} & L_{13}\\ L_{12} & L'_{22} & L_{23}\\ L_{13} & L_{23} & L'_{33}\end{pmatrix}\begin{pmatrix}A'_{mn}\\ B'_{mn}\\ C'_{mn}\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}. \tag{10}$$

Matrix elements are given by the formulas:

$$L'_{11}=L_{11}-\frac{\rho_m h^3}{12}\omega_{mn}^2,$$
$$L'_{22}=L_{22}-\frac{\rho_m h^3}{12\omega_{mn}^2}, \tag{11}$$
$$L'_{33}=L_{33}-\rho_m h\omega_{mn}^2,$$

where:

$$L_{11}=D_{11}\lambda_m^2+D_{66}\lambda_n^2+k^s A_{55},$$
$$L_{12}=(D_{12}+D_{66})\lambda_m\lambda_n,$$
$$L_{13}=k^s A_{55}\lambda_m, \tag{12}$$
$$L_{22}=D_{66}\lambda_m^2+D_{22}\lambda_n^2+k^s A_{44},$$
$$L_{23}=k^s A_{44}\lambda_n,\ \ L_{33}=k^s A_{55}\lambda_m^2+\lambda_n^2,$$

where:

$$\lambda_m=\frac{m\pi}{a},\ \lambda_n=\frac{n\pi}{b}. \tag{13}$$

If the rotatory inertia terms are neglected then $L'_{11}=L_{11}, L'_{22}=L_{22}$, and we get:

$$\omega_{mn}^2=$$
$$=\frac{(QL_{33}+2L_{12}L_{23}L_{13}-L_{22}L_{13}^2-L_{11}L_{23}^2)}{\rho_m hQ}, \tag{14}$$
$$Q=L_{11}L_{22}-L_{12}{}^2.$$

Also applies:

$$A'_{mn}=\frac{L_{12}L_{23}-L_{22}L_{13}}{Q}C'_{mn},$$
$$B'_{mn}=\frac{L_{12}L_{13}-L_{11}L_{23}}{Q}C'_{mn}. \tag{15}$$

2.2. Modeling of Sandwich Plates and Numerical Solution

For the numerical solution the simply supported Isorighe panel 1000 with microwave zinc plated steel sheet and laminate facings was used. The computational model without steel sheet ridges was used for simplify. It was investigated how the mechanical properties of the sandwich plates are changing, when the material constants of the core are stricted. Panel length is 5250 mm, nominal width is 1000 mm. Thickness of the panel is 100 mm in the case of statical analysis and 80, 100 and 120 mm in the case of dynamical analysis. On the panel affects static uniform linear load with intensity of 100 kg/m^2 in bending plane.

Thickness of zinc plated steel sheet facings h_f is 0.5 mm. Material characteristics are: E_s = 210 GPa; ν_s = 0.3; ρ_s = 7850 kg/m^3.

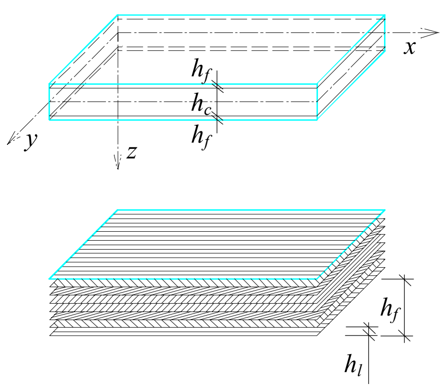

Fig. 2. *Layers of sandwich panel and fibres orientation of the laminate facing*

Thickness of laminate facings h_f is 1 mm, composed of eight layers of a symmetric laminate $[0/\pm45/90]_s$ and laminate with zero rotation angle of the fibres respectively. Thickness of each layer h_l is 0.125 mm. Geometry of sandwich panel is shown in Figure 2. It was considered the carbon fibres in epoxy matrix, while unidirectional laminate layer has characteristics [2]:

E_f = 230 GPa; E_m = 3 GPa; ν_f = 0.2; ν_m = 0.36; ξ = 0.6; ρ_k = 1508 kg/m^3.

Laminate properties determined by homogenization in the unit volume [3], [7], [9] are:

E_1 = 139.2245 GPa; E_2 = 12.2546 GPa; $G_{12} = G_{13}$ = 4.2753 GPa; G_{23} = 4.2751 GPa; $\nu_{12} = \nu_{13}$ = 0.2545; ν_{23} = 0.4333.

Effective material properties of the homogeneous laminate facings are:

E_x = 53.8762 GPa; E_y = 53.8762 GPa; G_{xy} = 20.3969 GPa; ν_{xy} = 0.3207.

Sandwich core, consisting of PUR foam, has material constants: E_{PUR} = 25 MPa; ν_{PUR} = 0.3; ρ_{PUR} = 100 kg/m^3.

Computational program MATLAB was used to calculate the material properties [3] of laminate facings. Numerical experiments were conducted through the COSMOS/M program. STAR module for solving linear static and DSTAR module for solving linear dynamic, were used for calculations. There were used finite elements of type SHELL4L. These are the 4 - node multi-layer quadrilateral elements with membrane and bending response; can be enter up to fifty layers.

3. Results and Discussions

Table 1 shows that displacements are depended on the type of used facings.

Table 1

Displacement depending on the type of facing

Facing		Displacement [mm]
Laminate	$[0/\pm45/90]_s$	37.627
	Homogeneous	41.077
	Zero angles	14.868

The sandwich panel with facings of symmetric laminate $[0/\pm45/90]_s$ demonstrated less value of displacement than sandwich panel with facings of homogeneous laminate. Difference between these two approaches is 9.17%.

Displacement of sandwich panel with zero angles of laminate fibres rotation was only 14.868 mm. This value confirms that there beam effect dominates due to the sandwich panel's dimensions.

The comparison of strains ε_x and ε_y (Figure 3) shows, that there are no significant variations in dealing with facings of laminate layers and their equivalent replacement. The values of stresses and strains in the x-axis are dominant, which result from the dimensions of the sandwich panel.

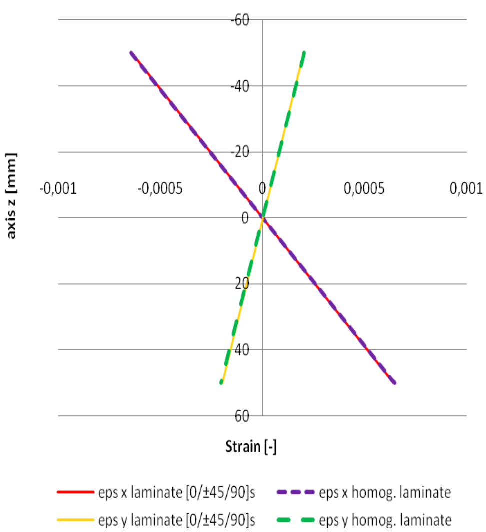

Fig. 3. *Distribution of the strains* ε_x *and* ε_y *over thickness of sandwich panel*

Distribution of the stresses σ_x over the thickness of sandwich panel is shown in Figure 4. Distribution of the stresses σ_x over the laminate layers of bottom sandwich's surface is shown in Figure 5. Values of the stresses are varied depending on rotation of the individual layers.

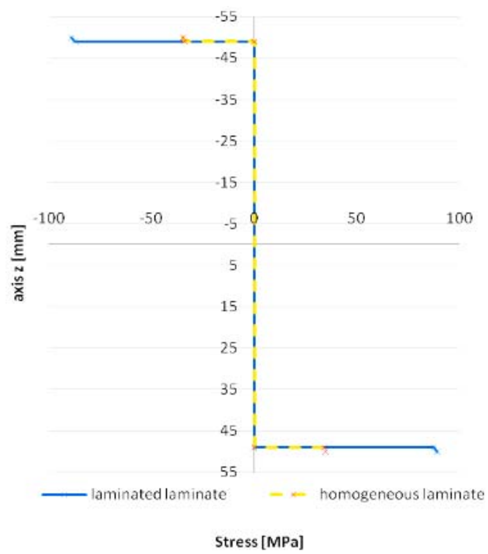

Fig. 4. *Distribution of the stresses* σ_x *over the thickness of sandwich panel*

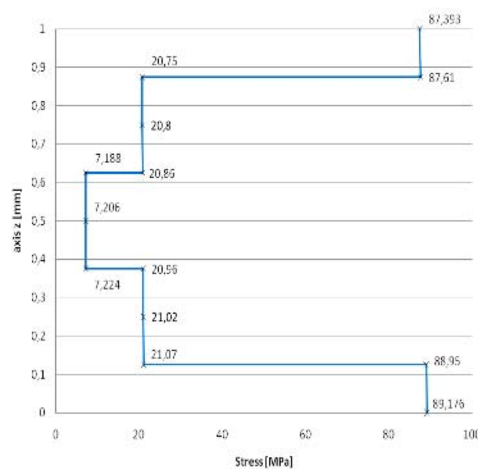

Fig. 5. *Distribution of the stresses* σ_x *over the laminate layers of bottom surface*

There were investigated natural frequencies and angular velocities of the first thirty natural mode shapes of vibration, depending on the type of facing used and the thickness of sandwich panel.

Natural mode shapes of sandwich panels vibrations with various thicknesses are shown in Figure 6.

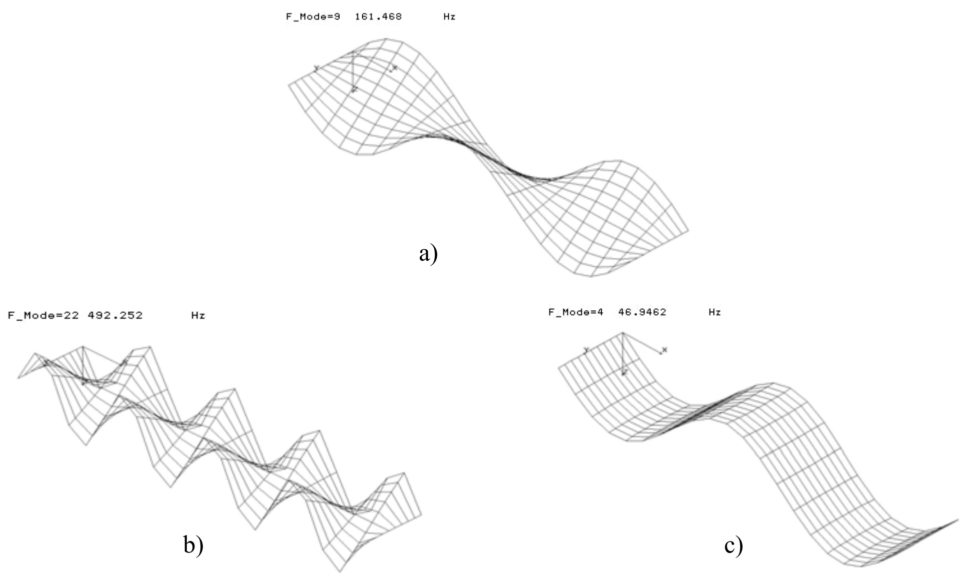

Fig. 6. *Natural mode shapes of vibration:*
a) *9th natural mode shape, laminate* [*0/±45/90*]$_s$*, 100 mm*; *b*) *22nd natural mode shape, laminate* [*0/±45/90*]$_s$*, 80 mm*; *c*) *4th natural mode shape, steel sheet, 80 mm*

Table 2 shows that change of sandwich panel's frequency, when its natural mode shape is changing, depends on the selected type of facing, while lower values are achieved in the steel sheet facings.

Change of natural angular velocity by various facings and natural mode shapes of sandwich panels is shown in Table 3.

Natural frequencies of sandwich panels Table 2

Natural mode shape	**Natural frequency *f* [1/sec = Hz]**								
Facing	Steel sheet			Laminate [0/±45/90]$_s$			Laminate (zero angles)		
	Panel thickness [mm]			Panel thickness [mm]			Panel thickness [mm]		
	80	100	120	80	100	120	80	100	120
1	7.7	8.9	9.9	7.1	8.2	9.1	11.3	13.0	14.5
2	25.6	28.9	31.7	28.4	32.5	35.9	25.1	28.5	31.4
3	29.0	31.8	34.0	45.9	51.1	55.0	44.4	50.3	55.1
4	46.9	52.0	55.9	634	66.9	62.4	62.3	61.7	57.8
5	56.6	62.0	66.1	72.7	71.8	78.8	66.6	70.0	76.3
6	68.5	74.9	75.0	94.2	104.3	111.9	96.4	107.1	115.2
7	81.9	79.0	79.9	111.4	124.6	130.6	115.1	127.5	131.3
8	83.8	89.5	95.4	146.7	140.1	135.0	151.1	140.0	136.7
9	89.4	97.1	102.9	152.1	161.5	172.3	163.6	177.9	187.4
10	104.6	114.1	121.3	170.8	186.3	173.7	181.8	197.5	207.2
11	109.3	118.2	124.8	204.8	188.5	201.3	206.6	221.1	207.5
12	124.9	135.6	143.7	240.2	223.4	236.4	224.5	222.6	207.7

Natural angular velocity of sandwich panels Table 3

Natural mode shape	**Natural angular velocity ω_{mn} [rad/sec]**								
Facing	Steel sheet			Laminate $[0/\pm45/90]_s$			Laminate (zero angles)		
	Panel thickness [mm]			Panel thickness [mm]			Panel thickness [mm]		
	80	100	120	80	100	120	80	100	120
1	48.2	55.9	62.8	44.7	51.4	57.4	71.3	81.7	90.9
2	161.1	182.2	199.4	178.4	203.9	226.1	157.5	178.9	197.1
3	182.3	200.1	213.7	188.7	320.8	345.7	278.9	315.8	346.3
4	294.9	326.9	351.9	398.6	420.7	392.1	390.9	387.6	363.2
5	355.5	389.7	415.6	456.9	451.4	495.4	418.8	440.1	479.7
6	430.3	471.1	471.5	592.1	655.4	703.7	605.8	673.3	724.0
7	514.5	496.7	502.1	699.7	783.1	820.4	723.1	801.0	825.0
8	526.8	562.6	599.2	922.0	880.1	848.4	949.4	879.7	858.6
9	561.5	610.1	646.7	955.8	1014.5	1082.5	1027.7	1118.0	1177.3
10	657.5	716.8	761.8	1073.5	1170.8	1091.4	1142.6	1240.8	1302.0
11	686.9	742.8	784.3	1286.6	1184.6	1265.1	1298.5	1389.4	130.8
12	784.8	852.2	902.9	1509.1	1403.9	1485.7	1410.7	1398.6	1305.1

Dependence of the angular velocity from its natural mode shape of vibration by various facing is in Figure 7 which shows, that the angular velocities are dependent on the type of sandwich panel facing, while the lowest values at the steel sheet facings are achieved.

When laminate facings are used, angular velocities are dependent from rotation of individual layers to the global coordinate system, while lower values were obtained for zero angles of rotation.

Change of angular velocity depending on its natural mode shape of sandwich panel vibration with a symmetric laminate facings and variable thickness of the core is shown in Figure 8, which shows that there are no significant discrepancies between the angular velocities when thickness of the core is changed.

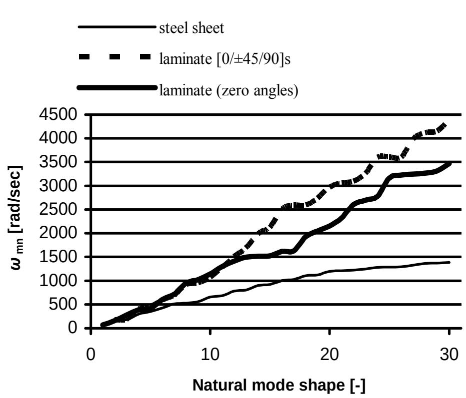

Fig. 7. *Angular velocities of the first thirty natural mode shapes of sandwich panels with 80 mm thickness*

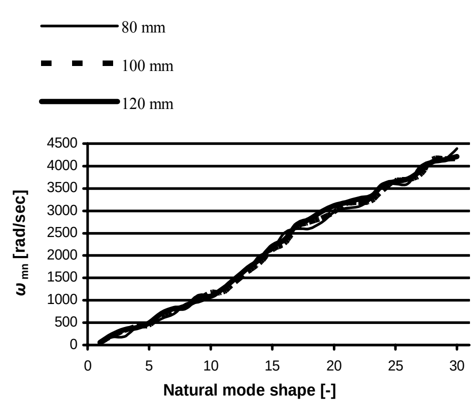

Fig. 8. *Angular velocities of the first thirty mode shapes of sandwich panels with symmetric laminate facings*

4. Conclusions

The mechanical properties of sandwich panels with laminate facings were investigated. There was used the Finite Element Analysis [5]. Type of fibres used in the matrix, their volume fraction and angle of rotation to the global coordinate system, affect the properties of the laminate. The sandwich plate made of the facings with laminate layers and homogeneous laminate were considered in the modeling. The values of displacements and stresses were more accurate when the facings of laminate layers were used counter to their equivalent replacement. Results of strains of the sandwich panels were similar in both approaches.

The facings of sandwich panel affect on its angular velocities and natural frequencies, while the lowest values were achieved by using of zinc plated steel sheet facings, what is related to their stiffness. Laminate facings affect the angular velocities and natural frequencies depending on the angle of rotation of the individual layers to the global coordinate system.

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