Bulletin of the *Transilvania* University of Braşov
Series I: Engineering Sciences • Vol. 4 (53) No. 2 - 2011

MODELS FOR TIME-VARIATION OF MECHANICAL STRENGTH FOR MATERIALS AND MACHINE PARTS

C. BEJAN[1]   V. SANDU[2]

Abstract: *The paper presents experimental results which demonstrate that both elasticity and fracture strength decrease during useful life. Experiments allow the determination of the qualitative characteristics of phenomena such as lowering elasticity and strength caused by intrinsic interference between the strength of metals and their usage stress. Based on experiments, the paper models the prediction with time of fracture probability and safety coefficients in two frequent practical situations: 1. Deterministic strength according to mean value and 2. Probabilistic strength with normal (Gaussian) distribution. In both cases it is considered an ergodic distribution of stress, variable between the extreme values adopted in the design phase.*

Key words: *reliability, strength and stress spectra, models of strength decrease, assessment of predictive failure rate.*

1. Ensuring Reliability through Design

In order to model the predictive reliability of mechanical parts it is required to know densities of distribution for stress and strengths of these components. In a classical perspective, operation without plastic deformation is ensured by a safety coefficient, a relatively conventional measure, used to compare two values that are conventional at their turn, mean stress and mean strength.

When safety coefficient values can be obtained empirically but with a high degree of confidence, the design based on them would provide optimal durability and reliability to the components.

For new systems that lack the empirical basis of assessing the safety coefficients, a modern approach of durability and reliability is based on probabilities of failure.

This approach has also the advantage to emphasize time-evolution of failure probability if it is known the function of initial strength decrease caused by phenomena as fatigue, wear, ageing etc. In their previous papers, the authors presented experiments on mechanical parts (springs, connecting rod screws etc.).

These experiments have proven that component operation leads to linear decrease in time of mechanical strength and a decrease of elasticity coefficient. It is presented in the following the mathematical model for the most important parameter of reliability: the intensity (the rate) of failure, $z(t)$.

[1] Road Vehicle Institute INAR, Braşov.
[2] Centre “High-tech Products for Automotives”, *Transilvania* University of Braşov.

2. Classification of Interference for Strength and Stress Spectra

Mathematic models for time-variation of reliability require, as a preliminary phase, the systematization of the two types of functions that determine the products' reliability: density distributions for strength and stress, then systematization of potential types of the two densities intersection.

By this systematization, it can be obtained an implicit classification for categories of general mathematical models regarding the evolution of reliability.

Probabilistic processing of waveforms characteristic to time-variation of stress for engine parts results in a stress spectrum having the coordinates f (Z)-Z or $f(\sigma)$-σ; where Z is the load and σ is the stress.

The density distribution for strength, with similar coordinates, can be obtained investigating fragments of the components (cut for test purpose - procedure that needs post-correction coefficients), or directly investigating the components, tests whose results are expressed probabilistic as well.

In case of a narrow dispersion band of results, stress, strength or both are deterministic. In case of a broad dispersion band, with clear emphasizing of a mean, of a mode, of superior and inferior limits - then stress, strength or both are probabilistic.

For characterization of the intersection between the stress spectrum and strength spectrum of components, it is assigned a letter to each of these two spectra, S for stress and R for strength. For the deterministic case it is assigned letter D, and for the probabilistic one, letter P. According to this convention, the types of possible intersections will be as follows:

- DD - deterministic stress spectrum and strength spectrum (Figure 1a);
- DP - deterministic stress and probabilistic strength (Figure 1b);
- PD - probabilistic stress and deterministic strength (Figure 1c);
- PP - probabilistic stress spectrum and strength spectrum (Figure 1d).

3. Study of Initial Strength Decrease in Time

Out of all aspects of predictive reliability, the way strength is decreasing in time is less clarified. Excepting the qualitative model of the Gassner diagram, there are no practical, direct approaches. Answers are still to come from development and finalization of research in mechanics of materials' fracture based on cumulative degradation models given by Palmgren-Langer-Miner (PLM), Freudenthal, Corten-Dolan [1], [2], [4], [5]; the methods used nowadays for the design of mechanical parts should consider more the elasticity of materials, it can be appreciated that in the near future these models are not likely to be applicable for the computation of provisional reliability using functions for the time-variation of strength. On the contrary, trends in reliability research - using more the results' justification based on failure repartitions as feed-back in the design phase - can endorse considerably the development of mathematical models for cumulative degradation, producing statistic rules of strength decrease with time, offering in this way an experimental basis for validation of the mechanical theories of materials' fracturing.

Literature on reliability recommends the following theoretical models for strength decrease in time - Table 1.

Considering the experience accumulated in the assessment of reliability indicators for mechanical parts, the model of linear decrease is the most common. Based on previous experiments with helical springs, connecting rod screws and counterweights as well as reliability results from machine tools [3], the authors suggest a combined model to be adopted for strengths decrease with time: an incipient parabolic zone followed by a linear one.

For DP and PD intersection types, fracture probability is zero until stress reaches the inferior strength, respectively strength remains greater than superior value of the stress; then the probability of fracture would increase following the variation rule of strength, respectively, of stress spectrum. For these types of intersection it is required to know the functions $\overline{R}(t)$ and $\overline{S}(t)$ (where upper bars indicate averaging), as well as the functions $R_i(t)$, $R_s(t)$ and $S_i(t)$ and $S_s(t)$, where indices i and s represent the minimum and, respectively, the maximum value.

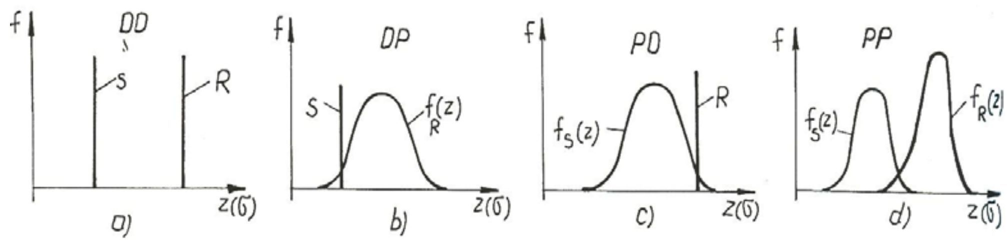

Fig. 1. *Classification of the intersections for distribution densities*

Theoretical models for strength decrease in time Table 1

Linear decrease	$R(t) = R_0 - \alpha t$, $R'(t) = -\alpha$,where α is the slope, expressed in units of tension or force divided by units of time;
Exponential decrease	$R(t) = R_0\, e^{-\alpha t}$, $R'(t) = -\alpha R_0 e^{-\alpha t}$, the decrease rate (the slope of the tangent to the curve) is variable;
Logarithmic decrease	Natural logarithm of the strength has a linear decrease with the logarithm of operation time or with the logarithm of number of cycles in operation;
Hyperbolic decrease	$R(t) = \dfrac{R_0 t_0}{t + t_0}$, where t_0 is an initialization parameter, dependent on safety coefficient;
Parabolic decrease	$R(t) = R_0 - \alpha t^2$, $R'(t) = -2\alpha t$ where $R'(t)$ is expressing the derivative of strength variation.

For all types of intersections, the problems regarding provisional reliability are: Is mean's variation is deterministic or probabilistic?; is the extremes' variation deterministic or probabilistic?

For any kind of repartition of stress or strength, there is a time t_1 until intersection occurs, a time with no failure. Decrease of strength until t_1 has no dependence on estimated limits of stress. In case of plastic materials, decrease of strength is due to ageing - if wear is concerned, strength can decrease as a consequence of physical-chemical phenomena that occur at the surface of the component. Regarding fatigue, strength can decrease as a consequence to modifications in the crystalline structure of materials being amplified when fatigue strength limits are overcome (helical springs, screws etc.). At t_1, when stress would interfere with strength, the variation of the strength would abruptly change its rate. This demonstrates the need to correct the Gassner diagram, paying attention to the mode of variation until t_1. From the point of view of provisional reliability it is more important to orient the assessment of reliability in operation towards the evaluation of this decrease of strength until t_1 than towards other indicators of reliability. It can be shown that, if the spectrum of strength is intersecting the spectrum of stress, the dispersion of strength is reduced. If the spectrum of stress is more intense, the dispersion of strength would be reduced more; these conclusions correspond to experimental observations showing that lower stress gives a broader spectrum of the strength, with broader dispersion, while higher stress results in narrow dispersion of strength.

4. Assessment of Provisional Failure Rate by Stress-Strength Convolution in PD Hypothesis

4.1. The failure rate

The model has an initial hypothesis that the number of failed products n_d is proportional to failure probability:

$$n_d = k\int_{S_i}^{S_S} f_S(Z)\cdot\Phi_R(Z,t)\,\mathrm{d}Z\,. \tag{1}$$

In equation (1) k is a constant of proportionality and $\Phi_R(Z, t)$ is the distribution function of the strength:

$$\Phi_R(Z,t) = \int_{R_{i(t)}}^{Z} f_R(Z)\,\mathrm{d}Z. \tag{2}$$

It should be emphasized that dynamic dependence on time is ensured by the minimum value of strength, R_i itself depending on time (decreasing during operation).

Introducing corrections for normalization, Λ_S, for stress and Λ_R, for strength, it yields:

$$n_d = k\Lambda_S^{-1}\ \Lambda_R^{-1}\int_{Si}^{Ss} f_S(Z)\Phi_R(Z,t)\,\mathrm{d}Z\,. \tag{3}$$

Experiments emphasize a finite domain for the mechanical strength of a component, within the limits R_i and R_s.

As $f_R(Z)$ should represent a density of distribution, the area under the curve between R_i and R_s should have a surface equal to 1 (Figure 2).

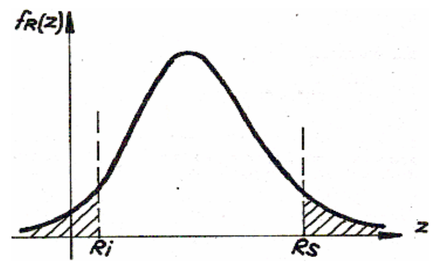

Fig. 2. *The density of distribution*

4.2. Particular case of stress-strength interference in PD hypothesis

If the dispersion of strength distribution is narrow, only the mean value of strength is needed in calculation. PD intersections are frequent, with normal stress distribution and strength that can be considered deterministic, by its mean value (decreasing in time). The selection of the determinist strength is justified as materials and components are produced within accurate technological processing. In the following presentation such an interference is considered, with Gaussian distribution spectrum for the strength, according to the formula:

$$f_S(Z) = \frac{1}{d_s\sqrt{2\pi}}\,\mathrm{e}^{\frac{-(Z-S)^2}{2\cdot d_s^2}}\,. \tag{4}$$

Failure probability will be calculated according to the formula:

$$P_D = \int_{R(t)}^{S_s} f_S(Z)\ \mathrm{d}Z, \tag{5}$$

where $R(t)$ is the time-variation of the strength, considered as deterministic throughout all the product lifespan:

$$n_d = kP_D = k\Lambda_S^{-1} \int_{R(t)}^{S_S} f_S(Z)\,\mathrm{d}Z \;... \qquad (6)$$

By differentiating the previous formula reported to time, it yields its derivative as follows:

$$\frac{\mathrm{d}n_d}{\mathrm{d}t} = k\Lambda_S^{-1}\left(\int_{R(t)}^{S_S} f_S(Z)\,\mathrm{d}Z\right)' = -k\Lambda_S^{-1} R'(t)\, f_S\big(R(t)\big), \qquad (7)$$

with $R'(t)$ is the temporal derivative of the function $R(t)$.

The number of products without failure, n_n, which are proportional to the complementary of failure probability, can be calculated with the formula:

$$n_n = k(1-P_D) = k\left(1-\Lambda_S^{-1}\int_{R(t)}^{S_S} f_S(Z)\,\mathrm{d}Z\right). \qquad (8)$$

It can be obtained the following formula of the failure rate for the PD case:

$$z(t) = -\frac{\Lambda_S^{-1} R'(t)\, f_S\big(R(t)\big)}{1-\Lambda_S^{-1}\int_{R(t)}^{S_S} f_S(Z)\,\mathrm{d}Z}. \qquad (9)$$

Using the Laplace function, Φ, for a simpler calculation, it results the failure rate for PD case and linear decrease of strength ($R(t) = R_0 - \alpha t$), according to the first line in Table 1, as indicated in relation (10).

For the current value of dispersion as the sixth part of the difference of maximum and minimum stress yields in $\frac{(S_S - S_i)}{6}$; it results $\frac{(S_i - \bar{S})}{d_S} = -3$, $(S_S - \bar{S}) = 3$ and $\Phi(-3) = 1 - \Phi(3) = 0.00135$ as indicated in relation (11):

$$z(t) = \frac{\alpha e^{-\frac{(R_0-\alpha t-\bar{S})}{2d_S^2}}}{d_s \cdot \sqrt{2\pi}\left[1-\Phi\left(\frac{S_i-\bar{S}}{d_S}\right) - 2\Phi\left(\frac{S_S-\bar{S}}{d_S}\right) + \Phi\left(\frac{R_0-\alpha t-\bar{S}}{d_S}\right)\right]}, \qquad (10)$$

$$z(t) = \frac{\alpha e^{-\frac{(R_0-\alpha t-\bar{S})^2}{2\cdot d^2{}_S}}}{d_S \cdot \sqrt{2\pi}\left[-0.00135 + \Phi\left(\frac{R_0-\alpha t-\bar{S}}{d_S}\right)\right]}. \qquad (11)$$

For the following numerical application having $1000 \le R_0 \le 10000$ (in daN), $\alpha = 0.1$ daN/hour, $\bar{S} = R_0/3$ it can be plotted the failure rate as presented in Figure 3, having as parameter inscribed the value of R_0.

All plotted curves show a dependency of type $z = at^\theta$ with a and θ as constants, defined by initial value R_0.

The general form of reliability distribution in case of PD intersection and Gauss probability P:

$$R(t) = \mathrm{e}^{-\int_0^t z(t)\,\mathrm{d}t} = \mathrm{e}^{-\int_0^t at^\theta\,\mathrm{d}t} = \mathrm{e}^{-\frac{at^{\theta+1}}{\theta+1}}. \qquad (12)$$

Last equation (12) indicates a Weibull distribution which is in accordance with the general findings that the reliability of mechanical components has a Weibull distribution.

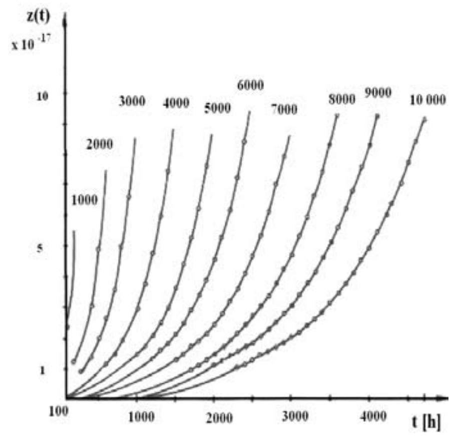

Fig. 3. *Failure rate for*
$1000 \leq R_0 \leq 10.000$ *daN*

5. Conclusions

According to models of provisional reliability of mechanical parts there were established relations for calculation of failure rate and its time variation, using the hypothesis of proportionality between number of failed products and failure rate.

For a linear time variation of strength and a Gaussian stress distribution (ergodic during product life span) it was determined the failure rate, basic parameter for reliability distribution. The results show that reliability law is Weibull type, according to experimental observations.

The calculations demonstrate that Weibull distribution is the statistic image of degradation produced by normal stress distribution and determinist (or a narrow dispersion) of in time decreasing strength.

This conclusion allows the elaboration of design procedures capable of early calculation for the quantitative parameters of reliability law.

References

1. Birolini, A.: *Quality and Reliability of Technical Systems. Theory, Practice, Management.* Berlin, Heidelberg. Springer-Verlag, 1997.
2. Birolini, A.: *Reliability Engineering. Theory and Practice*. Berlin, Heidelberg. Springer-Verlag, 2004.
3. Brătucu, Gh., Căpăţână, I.: *Researches Regarding the Reliability and Degradation of the Automation Equipment of Machine Tools.* In: Bulletin of the *Transilvania* University of Braşov (2007) Vol. 14 (49), Series I, p. 9-16.
4. Brooks, C.: *Failure Analysis of Engineering Materials.* New York. Mc Graw Hill, 2002.
5. Stephens, R.I., Fatemi, A., Stephens, R.R., et al.: *Metal Fatigue in Engineering.* USA. Wiley & Sons, 2001.