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MULTI-OBJECTIVE OPTIMIZATION OF THE REAR GUIDING LINKAGE OF A FORMULA STUDENT RACE CAR

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Abstract: *The paper approaches the multi-objective kinematic optimization of the rear suspension mechanism used for a formula student race car. The following steps are necessary for performing the optimization: parametrize the virtual model, defining the design variables and the design objectives for optimization, performing design studies for identifying the main design variables, and optimizing the model on the basis of these variables. The optimization criteria reffer to the variations of the wheel base, wheel track, induced deflection and camber angle. The study is performed by using the multi-body system (MBS) environment ADAMS of MSC Software, considering a DOE-based (Design of Experiments) investigation strategy.*

Key words: *multi-body system, rear suspension system, optimization.*

1. Introduction

It is known that, despite the "art-to-part" concept, which applies to the design, development and manufacturing of system components, the optimal system design isn't always based on optimal component design, but has at its base the interaction between form, fit, function and assembly of all parts of the system. Therefore, the best quality comes from using virtual prototyping methods, applied at system level. The decrease of the processing time, which allows the real time simulation, was shown by recent publications, which signal a growing interest in analysis methods for multi-body systems in view of the self-formulating algorithms [2], [4].

Before creating the physical prototype, engineers use such advanced programs to build virtual models of entire systems or subsystems, then to simulate their behaviors and to optimize the design. Crucial for the design and development stage is the simulation of the behavior under real operating conditions, and then the optimization of the form, fit and function characteristics of the system (product). This advanced simulation & optimization technique consists mainly in conceiving a detailed model and using it in a virtual experiment, in a similar way with the real case.

The main advantages are the reduced design time and cost, the smaller amount of product cycles, the smaller number of physical prototypes needed, as well as the possibility to perform virtual measurements in any point or area and for any parameter, and to optimize the system long before building the first hardware (physical) prototype.

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In this paper, virtual prototyping tools are used in the multi-objective kinematic optimization of the rear suspension mechanism used for a formula student race car. The virtual prototype of the suspension system is developed with the multi-body system (MBS) environment ADAMS of MSC Software.

2. The Virtual Prototyping Process

The CAD, MBS and FEA software solutions are frequently used in a virtual prototyping platform [1]. The MBS software is the main component of the platform, and it allows analyzing, optimizing, and simulating the mechanical system. The CAD software is used for creating the geometric (solid) model of the system. This model contains data about the mass & inertia properties of the rigid parts. The part geometry can be exported from CAD to MBS using standard format files, such as STEP or Parasolid.

The integration of the flexible components in the mechanical system model is possible by using the FEA software. Integrating flexibilities into model allows to capture inertial and compliance effects during simulations, to study deformations of the flexible components, and to predict loads with greater accuracy, therefore achieving more realistic results.

The steps to create a virtual prototype mirror the same steps to create a physical model, as follows: build (modeling parts, constrain the parts, create forces), test (measure characteristics, perform simulation, review animation), validate (import test data, superimpose test data), refine (add friction, define flexible parts, define command - control systems), optimize (add parametric, define design variables, define objective functions, perform design studies, perform optimization studies).

During the build phase, virtual prototypes are created of both the new product concept and any target products which may already exist in the market. The component solid models give the geometry and the mass properties of the bodies, whereas the structural and vibratory characteristics result from component finite element models.

One of the most important axioms for a successful virtual prototyping is to simulate as test. Testing of hardware prototypes has traditionally involved both lab tests and field tests in various configurations, which are very expensive. With virtual prototyping, it is enough to create virtual equivalents of the tests.

To validate the virtual prototype, the physical and virtual models are tested identically, using the same testing and instrumentation procedures. The results are compared, and design sensitivity analyses are performed on the virtual model to identify design parameters that have great influence on the performance results that do not correspond.

Refining the virtual prototype involves the fidelity to the model. Replacing the rigid components with flexible counterparts or adding frictions can improve the fidelity of the virtual model relative to the physical prototype.

The following steps are needed for the optimization of the virtual prototype: parameterizing the model, defining the design variables, defining the design objective for optimization, performing parametric studies, and optimizing the model.

The points which define the structural model, in fact the locations of the geometric constraints (i.e. the joints), are usually used for the parameterization of the mechanical systems. In this way, relationships within the model are created, so that when a point is changed, any other objects that depend on it will be updated.

Design variables represent elements in the model that allow creating independent parameters and to link modeling objects to them. In our case, the design variables represent the locations for the design points. In the parametric studies, the design variables are stretched through a range of values for finding the sensitivity of the overall system to these design variations. As a result, parametric study allows identifying the main design variables, with great influence on the design objective.

The problem of minimizing or maximizing a design objective over a selection of design variables, while satisfying various constraints on the design is called an optimization problem. A design objective is a numerical representation of the quality, efficiency, cost, or stability of the model. The optimum value of the design objective corresponds to the best design possible, which is achieved when the objective is minimized or maximized.

Part of the design process is to manipulate the unknowns (variables) in a design to arrive at a good design that satisfies all goals (objectives).

3. Case Study

In this paper, the virtual model of a complex mechanical system has been created to demonstrate the virtual prototyping capacities in a multi-objective optimization process. The application is performed for the rear wheel suspension mechanism of a Formula Student race car.

Because the suspension system is symmetrically disposed relative to the longitudinal axis of the vehicle, this paper proposes to optimize only a half of the mechanism. Therefore the optimization process was based on a single part of the rear axle. The model contains 6 bodies/ parts, as follows (Figure 1): 1 - upper control arm; 2 - lower control arm; 3 - wheel assembly; 4 - toe angle adjustment bar; 5 - chassis (car body).

In these terms, Figure 2 shows the connections between the bodies (i.e. the joints) in the right wheel suspension mechanism, as follows: A & B - spherical joints (bodies 1-5); C & D - spherical joints (2-5); E - spherical joint (1-3); F - spherical joint (2-3); G - spherical joint (3-4); H - spherical joint (4-5).

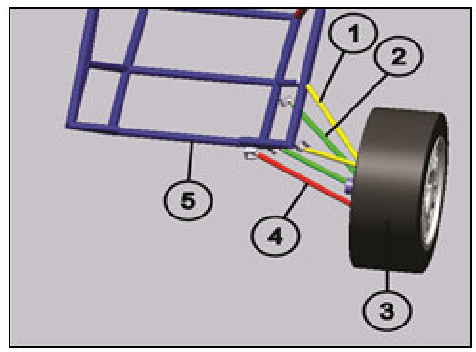

Fig.1. *Suspension bodies*

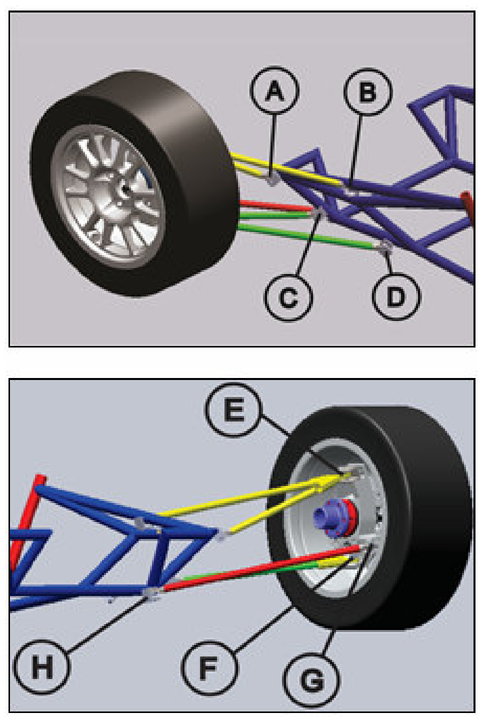

Fig. 2. *Suspension joints*

For the upper (1) and lower (2) control arms, the two spherical connections on car body (chassis) define, in fact, revolute joints. So, the virtual model of the suspension mechanism uses 8 points that control the locations of the joints.

Considering that the locations of the joints on the wheel assembly (wheel carrier) are established exclusively on constructive criteria, only the locations of the joints to the chassis and toe angle adjustment bar will be considered for the optimization process. Thus, there are 5 design points, in the global coordinates X, Y, Z for each point, and consequently 15 design variables, as follows:

$X_B \rightarrow DV\_1$, $Y_B \rightarrow DV\_2$, $Z_B \rightarrow DV\_3$;
$X_D \rightarrow DV\_4$, $Y_D \rightarrow DV\_5$, $Z_D \rightarrow DV\_6$;
$X_C \rightarrow DV\_7$, $Y_C \rightarrow DV\_8$, $Z_C \rightarrow DV\_9$;
$X_H \rightarrow DV\_10$, $Y_H \rightarrow DV\_11$, $Z_H \rightarrow DV\_12$;
$X_A \rightarrow DV\_13$, $Y_A \rightarrow DV\_14$, $Z_A \rightarrow DV\_15$.

The design variables allow to organize the critical parameters of the design into a concise list of values that can be easily modified, and to define independent parameters that can be linked to objects. In this way, a parametric model of the suspension mechanism has been created.

The optimization goal is to minimize the induced deflection, the wheel track deviation, the wheel base deviation and the camber angle. In the initial mechanism (before optimization) the time-history variations of these parameters are shown in Figures 3, 4, 5 and 6 with the following root mean squares (RMS): 0.62 mm - wheel base deviation; 6.73 mm - wheel track deviation; 0.39° - induced deflection, and 2.49° - camber angle. The optimization goal is to minimize these values.

4. Results and Conclusions

The optimization study is performed with ADAMS/Insight, part of the ADAMS suite of software, which is a powerful design-of-experiments (DOE) software.

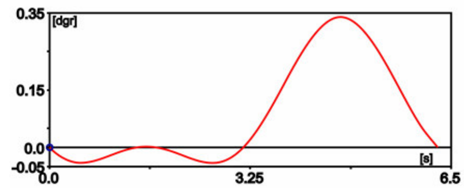

Fig. 3. *Initial induced deflection*

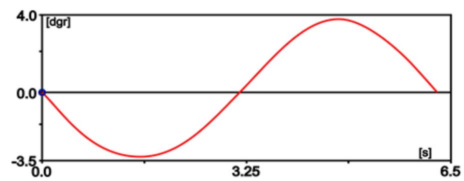

Fig. 4. *Initial camber angle*

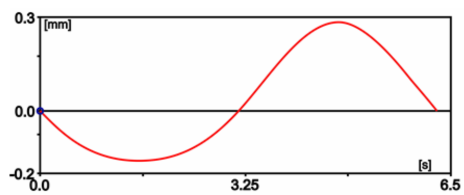

Fig. 5. *Initial wheel base deviation*

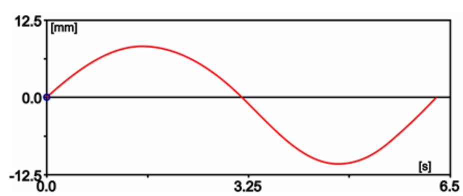

Fig. 6. *Initial wheel track deviation*

In order to run the optimization process in ADAMS/Insight (for the minimization of the induced deflection, wheel track deviation, wheel base deviation, and camber angle), the ADAMS/View model was exported as an experiment type file (*.xml). The optimization is performed in five steps: configuring the purpose of the experiment; setting the set of factors; planning a set of trials in which we vary the factor values from one trial to another; executing the runs and recording the performance of the suspension mechanism at each run; analyzing the changes in performance across the runs.

The runs are described by the design matrix, the matrix entries are the levels for each factor (design variable) per run. For each factor, there were defined the nominal (standard) value, and the variation field.

There are the following nominal values of the design variables, in the initial mechanism - before optimization (in [mm]): DV_1 = 320.02, DV_2 = 172.96, DV_3 = –1954; DV_4 = 241.12, DV_5 = 36.796, DV_6 = –1910.4; DV_7 = 238.63, DV_8 = 48.211, DV_9 = –2127.3, DV_10 = 178.82, DV_11 = 39.628, DV_12 = –2129.8; DV_13 = 321.48, DV_14 = 162.4, DV_15 = –2192.8. The variation field for each variable (factor) was set to [–10, +10] mm relative to the nominal value.

The investigation strategy used to create the design matrix is based on the DOE Screening technique, which allows identifying the factors and combinations of factors that most affect the behaviour of the suspension mechanism. The idea is to consider every factor that may potentially affect the response, and use a screening analysis to determine how much each contributes to the response. Due to the relatively high number of factors included in the optimization process (15 factors/ design variables), it was chosen the Plackett-Burman DOE technique [3], with 16 inputs in the design space.

The factors values combinations and the corresponding values of the design objectives (responses), which were obtained by successive analyses with the processing module ADAMS/Solver under the ADAMS/View interface, are presented in Table 1 (trials 1-5), Table 2 (trials 6-11) and Table 3 (trials 12-15). The so obtained work space is used to establish the relations between factors and responses.

For each response, there has been obtained the appropriate regression function, the goodness-of-fit summary being shown in Figure 7. R-squared (R2) is the proportion of total variability in the regression model data. Adjusted R-squared (R2adj) is based on the ratio of model mean square to total mean square. Regression significance (P) is defined as the probability that the regression coefficients are all zero. Range-to-variance ratio (R/V) measures how well a fitted regression might predict new values.

Table 1

	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**
DV_1	330.0	330.0	330.0	330.0	310.0
DV_2	163.0	183.0	183.0	183.0	183.0
DV_3	-1964.0	-1964.0	-1944.0	-1944.0	-1944.0
DV_4	231.1	231.1	231.1	251.1	251.1
DV_5	46.8	26.8	26.8	26.8	46.8
DV_6	-1920.4	-1900.4	-1920.4	-1920.4	-1920.4
DV_7	237.6	237.6	239.6	237.6	237.6
DV_8	58.2	38.2	38.2	58.2	38.2
DV_9	-2117.3	-2117.3	-2137.3	-2137.3	-2117.3
DV_10	168.8	188.8	188.8	168.8	168.8
DV_11	49.6	29.6	49.6	49.6	29.6
DV_12	-2139.8	-2119.8	-2139.8	-2119.8	-2119.8
DV_13	331.5	311.5	331.5	311.5	331.5
DV_14	172.4	172.4	152.4	172.4	152.4
DV_15	-2191.8	-2191.8	-2191.8	-2193.8	-2191.8
r_01	0.8	0.4	0.3	1.4	1.7
r_02	5.8	6.3	8.1	5.0	9.0
r_03	0.1	0.2	0.9	0.3	0.4
r_04	2.3	1.8	3.0	2.4	3.0

Table 2

	Trial 6	**Trial 7**	**Trial 8**	**Trial 9**	**Trial 10**	**Trial 11**
DV_1	330.0	310.0	330.0	330.0	310.0	310.0
DV_2	163.0	183.0	163.0	183.0	183.0	163.0
DV_3	-1944.0	-1964.0	-1944.0	-1964.0	-1944.0	-1944.0
DV_4	251.1	251.1	231.1	251.1	231.1	251.1
DV_5	46.8	46.8	46.8	26.8	46.8	26.8
DV_6	-1900.4	-1900.4	-1900.4	-1900.4	-1920.4	-1900.4
DV_7	237.6	239.6	239.6	239.6	239.6	237.6
DV_8	38.2	38.2	58.2	58.2	58.2	58.2
DV_9	-2137.3	-2137.3	-2137.3	-2117.3	-2117.3	-2117.3
DV_10	188.8	168.8	168.8	168.8	188.8	188.8
DV_11	29.6	49.6	29.6	29.6	29.6	49.6
DV_12	-2139.8	-2139.8	-2119.8	-2139.8	-2139.8	-2139.8
DV_13	331.5	311.5	311.5	331.5	311.5	311.5
DV_14	172.4	172.4	152.4	152.4	172.4	152.4
DV_15	-2193.8	-2191.8	-2191.8	-2193.8	-2193.8	-2191.8
r_01	0.0	0.5	0.3	0.3	0.1	1.1
r_02	6.9	7.0	6.3	6.7	5.9	5.8
r_03	0.3	0.5	0.5	0.1	0.7	0.2
r_04	1.7	1.8	2.4	3.6	2.2	3.0

Table 3

	Trial 12	Trial 13	Trial 14	Trial 15	Trial 16
DV_1	330.0	310.0	310.0	310.0	310.0
DV_2	163.0	183.0	163.0	163.0	163.0
DV_3	-1964.0	-1964.0	-1944.0	-1964.0	-1964.0
DV_4	251.1	231.1	231.1	251.1	231.1
DV_5	46.8	46.8	26.8	26.8	26.8
DV_6	-1920.4	-1900.4	-1900.4	-1920.4	-1920.4
DV_7	239.6	237.6	239.6	239.6	237.6
DV_8	38.2	58.2	38.2	58.2	38.2
DV_9	-2117.3	-2137.3	-2117.3	-2137.3	-2137.3
DV_10	188.8	188.8	168.8	188.8	168.8
DV_11	49.6	49.6	49.6	29.6	29.6
DV_12	-2119.8	-2119.8	-2119.8	-2119.8	-2139.8
DV_13	311.5	331.5	331.5	331.5	311.5
DV_14	152.4	152.4	172.4	172.4	152.4
DV_15	-2193.8	-2193.8	-2193.8	-2191.8	-2193.8
r_01	0.1	1.1	0.5	1.1	0.1
r_02	7.4	7.9	6.8	5.6	7.2
r_03	0.5	0.3	0.7	0.3	0.2
r_04	2.0	3.5	2.2	2.7	2.4

Goodness-of-fit for model "Model_01"

	r_01	r_02	r_03	r_04
R2	1	1	1	1
R2adj	1	1	1	1
P	0	0	0	0
R/V	1e+020	1e+020	1e+020	1e+020

Fig. 7. *The goodness-of-fit summary*

In the goodness-of-fit summary, the green bullets indicate that the fit criteria meet the fitting thresholds, so that the regression functions are useful (viable).

The optimization problem is a multi-objective one, which involves reaching the minimum values for the considered responses (the root mean squares of the wheel base - r_01; wheel track - r_02; induced deflection - r_03, and camber angle - r_04). The algorithm used to perform the optimization is OPTDES-GRG [5]. The factor values are adjusted so that the resulting responses come as closely as possible to the specified target values.

As ADAMS/Insight proceeds through the minimization, we will see the calculation converge; finally, the values of design variables will result in a simulation that meets the design requirements (Figure 8): r_01 = 0.005 mm; r_02 = 4.345 mm; r_03 = 0.0002°, r_04 = 2.048°.

	Minimum	Maximum	Value	Oper	Target
r_01	-1.0651	2.2925	0.0050659	MinTo	0
r_02	4.3213	9.1412	4.3454	MinTo	0
r_03	-0.26276	1.0492	0.00022637	MinTo	0
r_04	1.1431	3.851	2.0484	MinTo	0

Fig. 8. *Optimized values of the responses*

There can be observed a substantial reduction of the wheel base deviation and of the induce deflecton, but only a small decrease of the wheel track deviation and of the camber angle (there are necessary larger variation fields of the design variables for obtaining a greater reduction of these deviations). Under these circumstances, we can consider that the obtained variant is a good one, the variations being in the acceptable domains for the suspension systems of the Formula Student race cars.

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