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SOME CONSIDERATION CONCERNING THE MATHEMATICAL MODELING OF THE CAVITATIONS IN HYDRODYNAMIC LUBRICATION

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Abstract

In this paper, we present some numerical results related to the properties of the cavitations flow thin lubricant film which occurs between two contact surfaces of an axial-symmetrical tribological system (called radial seal surfaces). First, in the frame of Reynolds' model assumptions we get the homogenized equation generalized for the case when moving contact surfaces (rotor) has roughness properties and an arbitrary oscillatory motion around the parallel position with the other contact surface. Starting from the homogenized equation, we study the dynamics of free boundary of the domains where the cavitations flow occurs.

The first our objective is study the flow of a viscous Newtonian incompressible fluid film which occurs between two non-parallel surfaces, denoted S_2 (the rotor) and S_1 (the stator), which define a tribological system with axial symmetry, known as radial faces seals. The non-parallel character of the two seals is induced by the action of external perturbations (the most frequent being those of the vibration type). Due to these perturbations we shall have a supplementary movement of the rotor, similar to the movement of a rigid body to a fixed point.

The second our objective is the study of the non-parallelism of the two contact surfaces which induces the drastically modifications in the pressure field on the stator surface and consequently, the breakages of the lubricant film can appear on different zones of this.

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1 Indroduction

Moreover, the phenomena of occurrence growth and collapse of the lubricant film cavitations are very complex and in the cavitations zones the fluid changes its state passing into a mixture of gases and fluid vapours.

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In the classical theory of lubrication the Reynolds model describes the hydrodynamic behaviour of a film fluid between two seals of a tribological system (see Figure 1).

The Reynolds equation is obtained from the equations of Navier-Stokes and is written under the form [2], [8].

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial y}\right)=$$
$$=\frac{\partial}{\partial x}\left(\frac{\rho\left(V_1^x+V_2^x\right)h}{2}\right)+\frac{\partial}{\partial y}\left(\frac{\rho\left(V_1^y+V_2^y\right)h}{2}\right)+\frac{\partial}{\partial t}\left(\rho h\right), \tag{1}$$

where ρ is the density and μ the dynamical viscosity of the fluid. And obviously, according to the geometry of the system we have to specify the boundary conditions.

The field of pressure represents the unknown in the Reynolds equation and since the thickness of the fluid is small as compared to the representative length, we consider that the unknown is a function only of $M_1(x, y) \in S_1$ and of the time $t \in (0, +\infty)$.

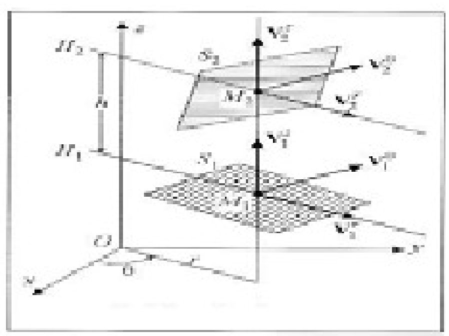


Figure 1: Geometrical representation of a generic tribological system

However, the Reynolds modelling does not take into account cavitations phenomena, which is defined as the rupture of the continuous film due to the formation of gas bubbles and makes the Reynolds equation no longer valid in the cavitations area. On the other hand, the cavitations may determine changes in the condition of the fluid, liquid or gas, or in the dry contact zones [4].

To investigate the cavitations flow, was taken into consideration different models such as: Sommerfield's model, Gümbel's model, Reynolds' model, the model of Floberg-Jakobsson-Olsson and the model of Elrod-Adams [3]. Only such, we can study the erosion of the surfaces in contact, the instability of the system determined by the high values of the pressure etc.

But, these models have been applied mainly to journal bearing case, while for radial seals faces case were obtained fewer significant results. For this reason, the main objective of our paper is coupling the three major problems of the hydrodynamic lubrication theory for the case of the radial seal faces: the cavitations phenomena, the effect of surfaces roughness and misalignment of the two contact surfaces.

2 Statement of Reynolds' model in the radial face seals case

In the case of an axial-symmetrical tribological system (see Figure 2) with a fixed S_1 surface (the stator) and another mobile S_2 (the rotor), we denote by h the distance between the two surfaces.

When the surfaces S_1 and S_2 are parallel we can have $h = const.$ or $h = l\,(t)$, $t > 0$, in the case of a vertical translation of the surface S_2 produced by the pressure variation, i.e. the squeeze effect.

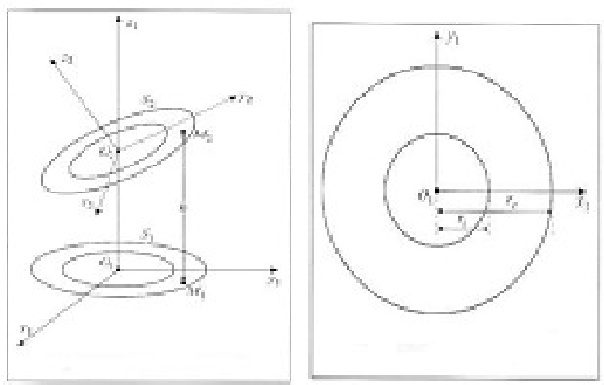


Figure 2: (a) Geometrical representation; (b) Horizontal projection

In this case the Reynolds equation (1) comes to

$$\frac{\partial}{\partial r}\left[rh^3\frac{\partial p}{\partial r}\right]+\frac{\partial}{\partial \theta}\left[\frac{h^3}{r}\frac{\partial p}{\partial \theta}\right]=6\mu h^2\left[\frac{\partial}{\partial r}\left(\frac{r}{h}V^r\right)+\frac{\partial}{\partial \theta}\left(\frac{V^\theta}{h}\right)\right], \tag{2}$$

where V^r is the radial velocity of S_2, V^θ the angular velocity, for $\forall\,(r,\theta)\in(r_i,r_e)\times(0,2\pi)$, $t>0$.

The boundary conditions are the following:

$$\begin{cases} P\,|_{r=r_i} = p_i\,(\theta,t)\,, \\ P\,|_{r=r_e} = p_e\,(\theta,t)\,, \end{cases} \tag{3}$$

where $p_i\,(\theta,t)$ et $p_e\,(\theta,t)$ are given pressures (problem data).

If the surface S_2 has an oscillatory movement like a rigid body movement with a fixed point, which is described by the Euler angles (ψ,φ,γ), (Figure 3), then the distance between the two surfaces S_1 and S_2 will be a function $h = h\,(r,\theta,t)$ written in the following exactly form [10]

$$h\,(r,\theta,t) = l\,(t) + r\,|\sin\left(\theta-\psi\,(t)-\omega t\right)|\tan\gamma\,(t)\,, \tag{4}$$

with r, θ polar coordinates and $l\,(t)$ the variation of S_2 on the vertically axis.

In this case the radial velocity will take the following form:

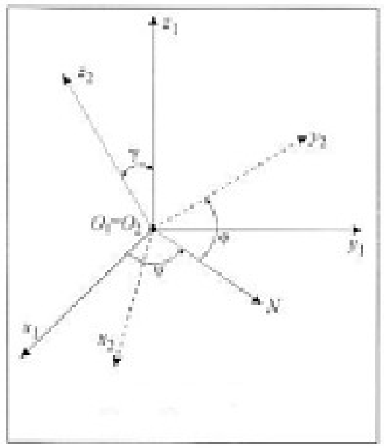


Figure 3: Euler angles

$$V^r = \left[-\dot{\gamma}\sin(\theta+\varphi) + \dot{\psi}\sin\gamma\cos(\theta+\varphi)\right]h, \tag{5}$$

and the angular one will be

$$V^\theta = \left(\omega + \dot{\varphi} + \dot{\psi}\cos\gamma\right)r - \left[-\dot{\gamma}\cos(\theta+\varphi) + \dot{\psi}\sin\gamma\sin(\theta+\varphi)\right]h, \tag{6}$$

where φ, ψ, γ are considered as functions of t.

Their derivatives as compared with times $\dot{\varphi}$, $\dot{\psi}$, $\dot{\gamma}$ are angular velocities. The velocity of rotation ω of the surface S_2 is constant and is considered as a characteristic of tightness.

In these hypotheses, the equation (2) takes the form

$$\frac{\partial}{\partial r}\left[rh^3\frac{\partial p}{\partial r}\right] + \frac{\partial}{\partial\theta}\left[\frac{h^3}{r}\frac{\partial p}{\partial\theta}\right] = 6\mu V(t)\, r\cos(\theta-\psi-\omega t)\tan\gamma, \tag{7}$$

with

$$V(t) = \omega + \dot{\varphi} + \dot{\psi}\cos\gamma.$$

For same radial seal faces case in [7] it use in place of Eq.(7) the following form of Reynolds' equation, write with our notations,

$$\frac{\partial}{\partial r}\left[rh^3\frac{\partial p}{\partial r}\right] + \frac{\partial}{\partial\theta}\left[\frac{h^3}{r}\frac{\partial p}{\partial\theta}\right] = 6\mu r\left(\omega\frac{\partial h}{\partial\theta} + 2\frac{\partial h}{\partial t}\right), \tag{8}$$

where the film thickness is considerate as

$$h = l + \gamma r\cos\theta, \tag{9}$$

with $l = const$. Furthermore, using the narrow seal approximation [14] equation (7) is written

$$\frac{\partial}{\partial r}\left[rh^3\frac{\partial p}{\partial r}\right]=6\mu\omega\frac{\partial h}{\partial \theta} \tag{10}$$

with

$$\frac{\partial h}{\partial \theta}=-\frac{r_i+r_e}{2}\gamma\sin\theta=-r_m\gamma\sin\theta. \tag{11}$$

Under these hypotheses, the solution of the equation (10), which fulfils the boundary problems (3), is written as following

$$p\left(r,\theta,t\right)=\left(p_i-p_e\right)\frac{r_e-r}{r_i-r_e}\left\{1+\left[\left(\frac{h_i}{h}\right)^3\frac{h_i-h}{h_e+h_i}-1\right]\right\}+p_e, \tag{12}$$

obviously $h_e\left(\theta,t\right)=h\left(r_e,\omega,t\right)$ and $h_i\left(\theta,t\right)=h\left(r_i,\omega,t\right)$ are obtained through (9). The first term in the braces of the equation (12) represents the well-known axisymmetric solution for the parallel seal surfaces

$$p_{axiym.}\left(r,\theta,t\right)=\left(p_i-p_e\right)\frac{r_e-r}{r_i-r_e} \tag{13}$$

and the second term is the contribution of misalignment

$$p_{misalig.}\left(r,\theta,t\right)=\left(p_i-p_e\right)\frac{r_e-r}{r_i-r_e}\left[\left(\frac{h_i}{h}\right)^3\frac{h_i-h}{h_e+h_i}-1\right], \tag{14}$$

which is not axisymmetric and it depends on the gap between the contact surfaces.

In [10], the equation (7) with the boundary conditions(3) had been solved and was obtained a classical solution.

3 Homogenization of the problem

In this paragraph, we shall study the problem (7), (3) by means of the multiple scales method with the hypotheses that S_1 is a surface with roughness [8]. For small film thicknesses the surface roughness becomes important in the performance of the lubricated contact. Even the real stator surface is rough at a microscopic level, this will influenced strongly the contact conditions. The Reynolds equation still applies since the heights of the surface asperities are small compared to the spatial elongation. Treatment of the roughness of a real surface in a deterministic fashion is however beyond in the scope of today's computers. Therefore other approaches need to be employed in order to take the surface roughness into account.

For applied the multiple scales method, first we introduce the dimensionless polar radius

$$\bar{r}=\frac{r-r_i}{r_e-r_i}, \tag{15}$$

with $0<r_i<r_e$, the two radiuses of the annulus.

If we perform a change in the variables

$$\begin{cases} x_1 = \bar{r} \\ x_2 = \frac{\theta}{2\pi} \end{cases} \tag{16}$$

we have $\vec{x} := (x_1, x_2) \in Y = (0,1) \times (0,1)$.

Thus, the pressure, defined in the domain $\Omega = (r_i, r_e) \times (0, 2\pi)$, becomes

$$\tilde{p}(x_1, x_2, t) := p(r(x_1), \theta(x_2), t), \tag{17}$$

and the distance between the two surfaces becomes

$$\begin{aligned} \tilde{h}(x_1, x_2, t) &:= \tfrac{1}{r_e - r_i} h(r(x_1), \theta(x_2), t) = \\ &= \tfrac{l(t)}{r_e - r_i} + (x_1 + c)\left|\sin(2\pi x_2 - \psi - \omega t)\right| \tan\gamma \end{aligned} \tag{18}$$

where

$$c := \frac{r_i}{r_e - r_i}. \tag{19}$$

Under these hypotheses the Reynolds equation (7) takes the following form:

$$\begin{aligned} &\frac{\partial}{\partial x_1}\left[(x_1 + c)\tilde{h}^3 \frac{\partial \tilde{p}}{\partial x_1}\right] + \frac{1}{4\pi^2}\frac{\partial}{\partial x_2}\left[\frac{\tilde{h}^3}{(x_1 + c)}\frac{\partial \tilde{p}}{\partial x_2}\right] = \\ &= \frac{\mu}{\pi}\left[(x_1 + c)\left(\omega + \dot{\varphi} + \dot{\psi}\cos\gamma\right)\right]\frac{\partial \tilde{h}^3}{\partial x_2}. \end{aligned} \tag{20}$$

We introduce the denotation:

$$\begin{cases} a_1(\vec{x}, t) = (x_1 + c)\tilde{h}^3 \\ a_2(\vec{x}, t) = \frac{1}{4\pi^2}\frac{\tilde{h}^3}{(x_1 + c)} \end{cases} \tag{21}$$

or under vector form $\vec{a}(\vec{x}, t) = (a_1(\vec{x}, t), a_2(\vec{x}, t))$, and

$$\tilde{V}(t) = \frac{\mu}{\pi}\left(\omega + \dot{\varphi} + \dot{\psi}\cos\gamma\right). \tag{22}$$

With these denotations the equation (20) becomes

$$\frac{\partial}{\partial x_1}\left[a_1(\vec{x}, t)\frac{\partial \tilde{p}}{\partial x_1}\right] + \frac{\partial}{\partial x_2}\left[a_2(\vec{x}, t)\frac{\partial \tilde{p}}{\partial x_2}\right] = \tilde{V}(t)\frac{\partial}{\partial x_2}\left[a_2(\vec{x}, t)\right] \tag{23}$$

The roughness defaults, whose typical amplitude is given, can be modelled with the introduction of a small parameter ε, which denotes the typical spacing between two patterns, and $\varepsilon \to 0$. In this framework, gap functions become highly oscillating. It is assumed that the roughness is periodic with period ε. In other words, we consider the distance between the contact surfaces of the form

$$h_\varepsilon(\vec{x}) = h\left(\vec{x}, \frac{\vec{x}}{\varepsilon}\right), \tag{24}$$

and hence, the coefficients of the equation (7) became rapidly oscillating periodic functions of the local variable $\frac{\vec{x}}{\varepsilon}$ of the base cell $Y=(0,1)\times(0,1)$.

The multiple scales method will be used for the study of the equation (23). We shall seek $\tilde{p}^{\varepsilon}$ under the form of an asymptotic development:

$$\tilde{p}^{\varepsilon}=\tilde{p}^{0}\left(\vec{x},t\right)+\varepsilon\tilde{p}^{1}\left(\vec{x},\frac{\vec{x}}{\varepsilon},t\right)+\varepsilon^{2}\tilde{p}^{2}\left(\vec{x},\frac{\vec{x}}{\varepsilon},t\right)+\cdots. \tag{25}$$

with local variable definite by

$$\vec{y}=\frac{\vec{x}}{\varepsilon}, \tag{26}$$

where ε describe the roughness of the stator S_1 and $\varepsilon\to 0$.

The operators of the derivative are thus

$$\begin{cases}\frac{\partial}{\partial x_1}\to\frac{\partial}{\partial x_1}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_1},\\ \frac{\partial}{\partial x_2}\to\frac{\partial}{\partial x_2}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_2}.\end{cases} \tag{27}$$

If we denote by [5-6]

$$a_1^{\varepsilon}=a_1\left(\frac{\vec{x}}{\varepsilon},t\right),\ a_2^{\varepsilon}=a_2\left(\frac{\vec{x}}{\varepsilon},t\right) \tag{28}$$

in (23), we have

$$\begin{gathered}\left(\frac{\partial}{\partial x_1}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_1}\right)\left\{a_1^{\varepsilon}\left[\left(\frac{\partial\tilde{p}^0}{\partial x_1}+\frac{\partial\tilde{p}^1}{\partial y_1}\right)+\varepsilon\left(\frac{\partial\tilde{p}^1}{\partial x_1}+\frac{\partial\tilde{p}^2}{\partial y_1}\right)+\varepsilon^2\left(\frac{\partial\tilde{p}^2}{\partial x_1}+\frac{\partial\tilde{p}^3}{\partial y_1}\right)+\cdots\right]\right\}+\\ \left(\frac{\partial}{\partial x_2}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_2}\right)\left\{a_2^{\varepsilon}\left[\left(\frac{\partial\tilde{p}^0}{\partial x_2}+\frac{\partial\tilde{p}^1}{\partial y_2}\right)+\varepsilon\left(\frac{\partial\tilde{p}^1}{\partial x_2}+\frac{\partial\tilde{p}^2}{\partial y_2}\right)+\varepsilon^2\left(\frac{\partial\tilde{p}^2}{\partial x_2}+\frac{\partial\tilde{p}^3}{\partial y_2}\right)+\cdots\right]\right\}=\\ =\tilde{V}\left(t\right)\left(\frac{\partial}{\partial x_2}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_2}\right)\left[a_2\left(\vec{x},t\right)\right]=\frac{1}{\varepsilon}\tilde{V}\left(t\right)\frac{\partial a_2^{\varepsilon}}{\partial y_2}\end{gathered}$$

At the order ε^{-1} we obtain the equation

$$\frac{\partial}{\partial y_1}\left(a_1^{\varepsilon}\left(\frac{\partial\tilde{p}^0}{\partial x_1}+\frac{\partial\tilde{p}^1}{\partial y_1}\right)\right)+\frac{\partial}{\partial y_2}\left(a_2^{\varepsilon}\left(\frac{\partial\tilde{p}^0}{\partial x_2}+\frac{\partial\tilde{p}^1}{\partial y_2}\right)\right)=0 \tag{29}$$

which has the form

$-div_y\left(\vec{a}^{\varepsilon}\left(\vec{y},t\right)\cdot grad_y\tilde{p}^1\right)=div_y\left(\vec{a}^{\varepsilon}\left(\vec{y},t\right)\cdot grad_x\tilde{p}^0\right).$

With $\tilde{p}^0$ known, we seek the solution $\tilde{p}^1\in H^1\left(Y\right)$ of the local problem (29) under the form

$$\tilde{p}^1\left(\vec{x},t,\vec{y}\right)=\chi_1\left(\vec{y}\right)\frac{\partial\tilde{p}_0}{\partial x_1}+\chi_2\left(\vec{y}\right)\frac{\partial\tilde{p}_0}{\partial x_2}, \tag{30}$$

where χ_1 et χ_2 (the correctors) are the solutions of the equation

$$-\frac{\partial}{\partial y_i}\left(a_j^{\varepsilon}\frac{\partial\chi_k}{\partial y_j}\right)=\frac{\partial a_k^{\varepsilon}}{\partial y_k},\ k=1,2, \tag{31}$$

with $\chi_k\in H^1\left(Y\right)$ and of zero, $\langle\chi_k\rangle=\frac{1}{|Y|}\int_Y\chi_k\left(\vec{y}\right)d\vec{y}=0,\ \ k=1,2.$

If we introduce the functions $w_k \in H^1(Y)$, Y- periodic and of zero mean$\langle w_k \rangle = 0$, then the functions

$$w_k = -\chi_k + y_k,\ k = 1, 2 \tag{32}$$

are the solutions of the equations

$$div_y\left(\vec{a}^{\varepsilon}(\vec{y}, t) \cdot grad_y w\right) = 0. \tag{33}$$

Similar to the classical theory of homogenization [5], [6], [12], the homogenized coefficients are defined

$$\begin{cases} q_1^0 = \langle a_1^{\varepsilon} \rangle + \left\langle a_1^{\varepsilon} \frac{\partial \chi_1}{\partial y_1} \right\rangle \\ q_2^0 = \langle a_2^{\varepsilon} \rangle + \left\langle a_2^{\varepsilon} \frac{\partial \chi_2}{\partial y_2} \right\rangle \end{cases} \tag{34}$$

resulting in the homogenized equation

$$-\sum_{i=2}^{2} \frac{\partial}{\partial x_i}\left(q_i^0 \frac{\partial \tilde{p}^0}{\partial x_i}\right) = f(\vec{x}, t), \tag{35}$$

with $\tilde{p}^0 \in H_0^1(\Omega)$.

In the axial-symmetrical case, with $l(t) \equiv 0$ in (18), the solution of the equation (33) is obtained under the form [10]

$$w(\vec{y}) = A + B\frac{1}{\tilde{h}_{\varepsilon}^2} \tag{36}$$

with $A, B \in \mathbb{R}$ such that $\langle w \rangle = 0$. The last condition is hold if and only if $A = 0$. Then we can take

$$\begin{cases} \chi_1(\vec{y}, t) = \frac{1}{\tilde{h}_{\varepsilon}^2} + y_1, \\ \chi_2(\vec{y}, t) = \frac{1}{\tilde{h}_{\varepsilon}^2} + y_2. \end{cases} \tag{37}$$

With (37) we can calculate the homogenized coefficients

$$\begin{cases} q_1^0 = \frac{2}{5\pi}\tan^3\gamma\left[(1+c)^5 - c^5\right]\left[\frac{1}{3}\cos^3(\psi + \omega t) - \cos(\psi + \omega t)\right], \\ q_2^0 = \frac{1}{6\pi^3}\tan^3\gamma\left[(1+c)^3 - c^3\right]\left[\frac{1}{3}\cos^3(\psi + \omega t) - \cos(\psi + \omega t)\right]. \end{cases} \tag{38}$$

It may be noted that in the equation (35), with the coefficients resulted from (38), we can simplify by $\tan\gamma$ in the particularly case of the exterior forces absence, and thus the right-hand member does not depend anymore on this parameter.

If we consider the case when S_1 and S_2 are parallel, i.e. $\gamma = \varphi = \psi = 0$, then the homogenized equation (35) becomes

$$\frac{1+2c}{2\pi}\sin^2\left(\frac{\omega t}{2}\right)\frac{\partial^2 \tilde{p}^0}{\partial x_1^2} = 6\mu\omega(x_1 + c)\cos(2\pi x_2 - \omega t). \tag{39}$$

The solution of (39) has the form

$$\tilde{p}^0(\vec{x},t) = \frac{12\pi\omega\mu}{(1+2c)\sin^2\left(\frac{\omega t}{2}\right)}\left[\frac{1}{6}(x_1+c)^3 + C_1x_1 + C_2\right]\cos(2\pi x_2 - \omega t), \tag{40}$$

with $C_1, C_2 \in \mathbb{R}$, such that:

$$\begin{cases} \tilde{p}^0(\vec{x},t)\,|_{x_1=0} = C_2 + \frac{1}{6}c^3 = 0 \\ \tilde{p}^0(\vec{x},t)\,|_{x_1=1} = C_1 + C_2 + \frac{1}{6}(1+c)^3 = 0 \end{cases} \tag{41}$$

for $\forall x_2 \in (0,1)$ and $\forall t > 0$. We obtain

$$\begin{cases} C_1 = -\frac{1}{6}c\left[3c^2+3c+1\right], \\ C_2 = -\frac{1}{6}c^3. \end{cases} \tag{42}$$

4 Some numerical results for equation (39)

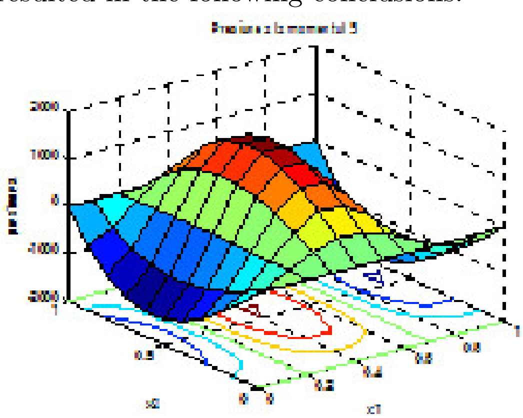
In this paragraph we present the numerical solution of equation (39) in the case $l(t) = 0$, with the affine term given under the form [11]:

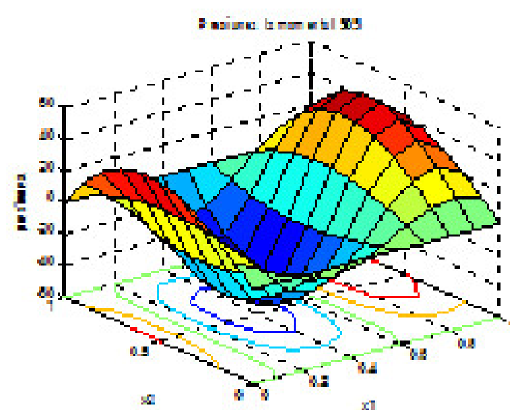
$$f(\vec{x},t) = 6\mu\omega(x_1+c)\cos(2\pi x_2 - \omega t), \tag{43}$$

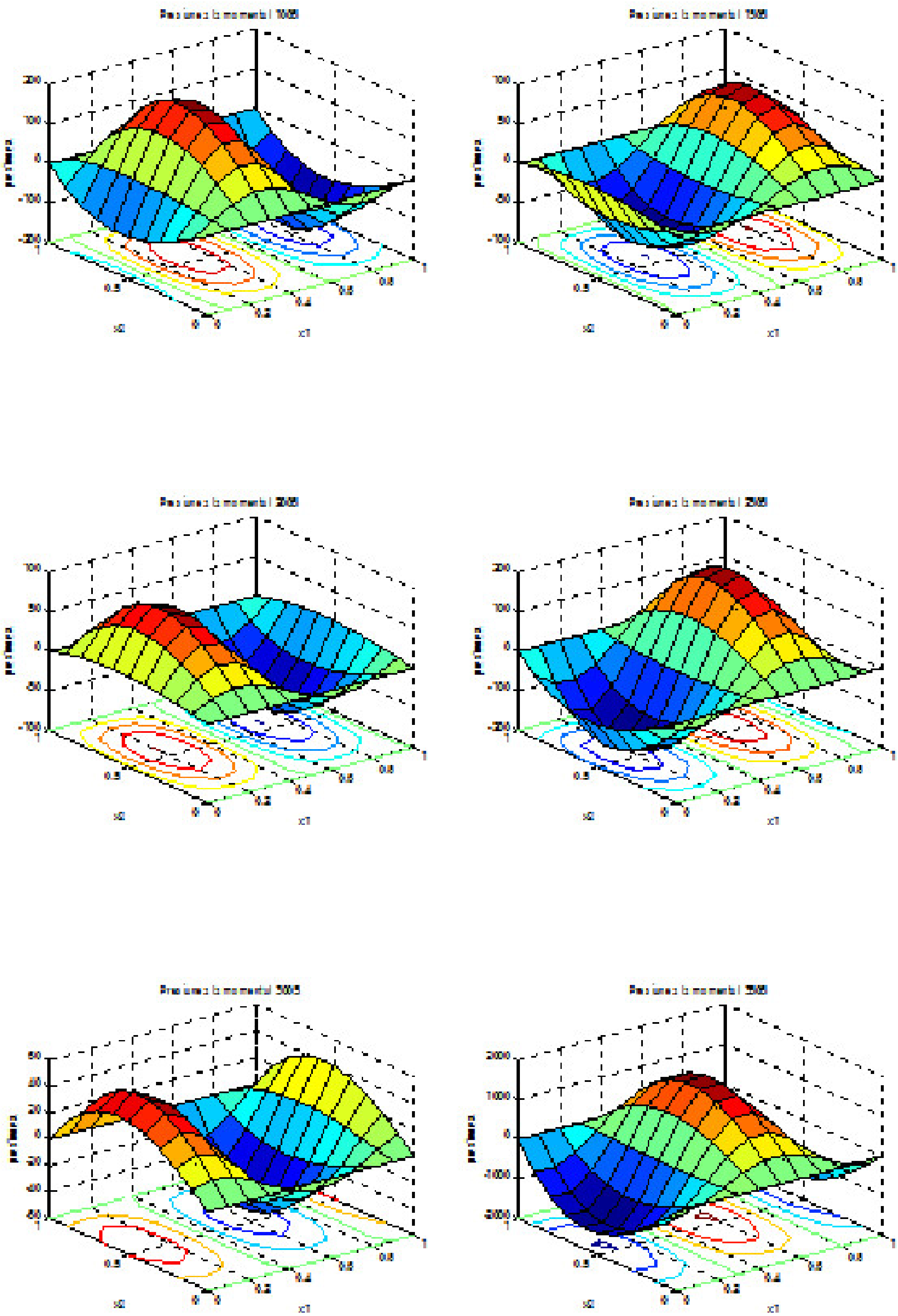
and the coefficients given in the relation (38), and the boundary conditions imposed by (41).

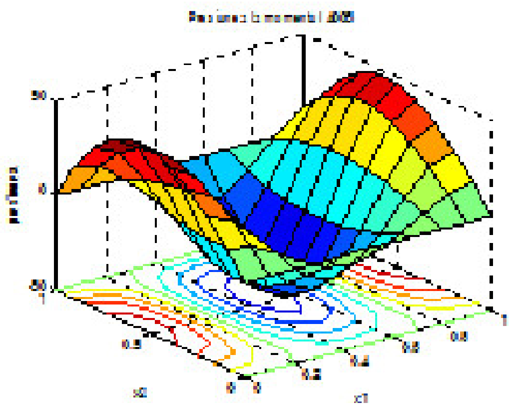
The numerical results had been obtained by means of the *MATLAB*. In this paper we present only a succession of the graphical representation of the (40) in the case of the tightness geometries given by $r_e = 10$, $r_i = 1$, $c = \frac{1}{9}$ and for the angular velocity of the rotor $\omega = 500$ rot/min, keeping constant the variation step for the time $\Delta t = 500$ sec over the range 5 sec $\leq t \leq 5.000$ sec.

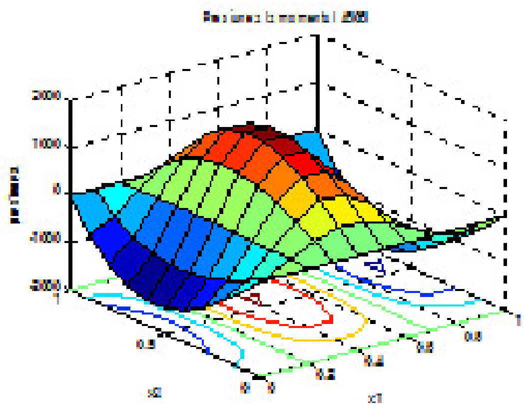
The numerical simulations effected by us, schematically described in the present paper, resulted in the following conclusions.











First, the homogenization of the equation of Reynolds leads to a more realist model for the cavitations phenomenon of the lubricant fluid film, as we observe even in the case when the contact surfaces are parallel, by the directly comparison between equations (12) and (39). In fact, when the contact surfaces are parallel, the pressure field as solution of the equation (39) has negative values, which under the incidence of the hypotheses of the Sommerfield model, i.e. $\tilde{p}^0(\vec{x}, t) < 0$ with $\vec{x} \in \Omega$ and $t > 0$, and it defines the zones where the cavitations phenomenon occurs and implicitly, the free boundary is described by the equations $\tilde{p}^0(\vec{x}, t) = 0$.

Secondly, the numerical simulations showed a strong dependency of the cavitations phenomenon in relation to the geometrical parameter c, and the influence of the values of rotational velocity ω is expressed rather by the definition of the cavitations sub-domains for a real tightness.

Thirdly, the numerical results point out the property of periodicity in relation to the time for the dynamics of the cavitations zone both for its geometrical form and for its location on the based cell. It is obvious that this conclusion is one purely mathematically, since it does not take into consideration the random motions of the rotor, which are by vibration type around the parallel position, as in the homogenized equation (39), a simulation of the behavior for different real mechanical systems cannot be obtained.

As a conclusion, the Reynolds model based on the equation (39) is adequate for describing the cavitations lubricant films flow, reason for which we shall continue the research in this domain.

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