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ADAPTIVE PI CONTROLLER DESIGN TO CONTROL A MASS - DAMPER - SPRING PROCESS

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Abstract: *In this paper, the authors have been applied the adaptive control to a mass-damper-spring. During the whole design process only parametric disturbances have been considered. Based on the uncertainty parameters (mass, damping constant, and spring) and by using a gradient method (the MIT rule), a PI adaptive controller is proposed and designed. The used gradient method (i.e. the MIT rule) allowed for the studied parameters to be varied in a predefined range. The whole design, as well as the experimental results, was done in Matlab/Simulink.*

Key words: *mass - damper - spring, MRAS, model reference, adaptive control, MIT rule.*

1. Introduction

In general, the term of adapting to something can be explained as behavior changes due to direct reactions for given circumstances. Based on this broad definition, an adaptive controller can be defined as the controller's ability to modify its behavior in response to any changes of the process dynamics that have an impact on him.

Historically speaking, the whole progress in the microelectronics area was a real stimulus for plenty of adaptive control experiments developed in special laboratories and industry. The results were not left waiting - in the early 80s, adaptive controllers were commercially launched and heavily used in industry. The key for having such a success was given by the very narrow definition for adaptive control: to easily adapt to any change of the process dynamics and, eventually, to gain the needed control through the whole process.

Due to the large scale of adaptive controllers, in this paper the studied case is related to mass-damper-spring. One of the most important characteristics imposed to a good adaptive controller design is the capability to ensure the system stability against any of the uncertainty parameters. Considering such premises as a valid starting point (n.b. it is known that the values of uncertainty parameters vary frequently and significantly), in this paper an adaptive control strategy is proposed and, based on it, a model of system uncertainty can be specified further by a designer.

When dealing with uncertainty parameter is recommended and considered a good practice for correctly design managing, to use adaptive control. Therefore, searching

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for a proper controller is adequate in most cases. For a controller to be considered fit to its purpose, one rule has to be accomplished - such that in case of a closed loop system, a certain adaptive stability needs to be achieved. As a consequence, one of the main approaches applied in adaptive control is Model Reference Adaptive System (MRAS).

Adaptive control is a technique that provides automatic and real time adjustment for a controller. Such an adjustment is performed when the studied process presents unknown but constant parameters (in this case, the design of adaptive control should come up with an automatic tuning procedure applicable to all these parameters in a closed loop system), or unpredictably changing in time parameters (in this case, for the system performances to be maintained, adaptive control of the control system should be used). The goal is to maintain system performances of a controller to its imposed thresholds [1], [4], [5].

Although the original scheme for MRAS, which was proposed by Whitaker in 1958, was introduced for flight control, in the current study is applied to DC electrical drive controlling (Figure 1) [1].

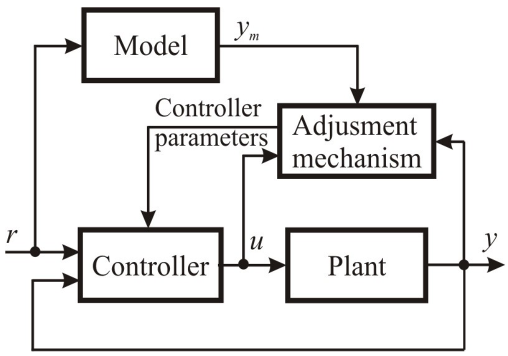

Fig. 1. *General block diagram of MRAS*

The gradient method mentioned above is used to design and simulate a MRAS system.

2. Model Reference Adaptive Control (MRAC)

It is considered a starting point the desired behavior of a given process that can be described. It can be achieved by using a model reference. Particularly, a linear time-invariant system (LTI) is representing the process, while the model reference is implemented as MRAC. The aforementioned process is driven by its input reference and has associated the transfer function $G_m(s)$.

MRAS was derived from continuous systems and has an inner loop and an outer loop. The process itself and classical feedback are included in the inner loop, while the outer loop is only used to adjust controller parameters.

A complex process in designing of the transfer function for model reference ($G_m(s)$) can be outlined by following the behavior of 3 (three) important signals (input, output, and error). The input signal is based on a given reference input signal $r(t)$ and the output signal $y_m(t)$ is represented by the system desired response.

An important goal is to succeed in diminishing the gap between system output and reference model. This gap is considered the error signal $e(t)$ and its size depends on the chosen model reference, the process $y(t)$ - which must follow the output signal, and the command signal. It is said that a perfect model can be achieved when, for all command signals, the error signal is reduced to a null value [2], [3], [5], [7].

In the particular case of MRAC, all parameters can be adjusted either by using a gradient method, or by applying a stability theory.

2.1. MRAS Designing by Using the Gradient Method

The gradient method, also named the MIT rule due to fact that was developed by the

Instrumentation laboratory at Massachusetts Institute of Technology (MIT), is one of the two aforementioned approaches for MRAC discussed in this paper. The associated equation to MIT rule is:

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J}{\partial\theta} = -\gamma e\frac{\partial e}{\partial\theta}, \qquad (1)$$

where: θ - is the controller parameter; *e* - is the error between the process and the model outputs; γ - is the adaptation gain; $\frac{\partial e}{\partial\theta}$ - is the system's sensitivity derivative.

To understand better the MIT rule and its purposes, a few explanations are mandatory. A first assumption is that for the examined closed loop system, the given controller has one single adjustable parameter θ. The next assumption is to determine the error (*e*) between the process output ($y(t)$) and model reference output ($y_m(t)$).

To succeed in minimizing of the loss function $J(\theta) = \frac{1}{2}e$ [4], [6], the parameter θ must be adjusted. Changing the parameters in a way of having negative gradient forces the function *J* to be small. One important aspect is that loss function is randomly chosen. Supposing that loss function is $J(\theta) = |e|$, then the adjustment factor depends, among other variables, on signum function, too:

$$\frac{d\theta}{dt} = -\gamma\frac{\partial e}{\partial\theta}\operatorname{sign} e, \qquad (2)$$

3. Process Modelling of the Mass - Damper - Spring

A mass - damper - spring system can be described like in the Figure 2.

Using the second law of Newton, the system dynamic can be represented by the following second order differential Equation:

$$m\frac{d^2x(t)}{dt} + c\frac{dx(t)}{dt} + kx(t) = F(t), \qquad (3)$$

where *m* is the mass, *c* the damping constant, *k* the spring stiffness, $x(t) = y(t)$ the displacement, and $F(t) = u(t)$ the external force.

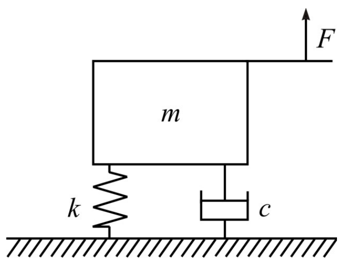

Fig. 2. *Mass-damper-spring process*

The block diagram of the process is represented in the Figure 3.

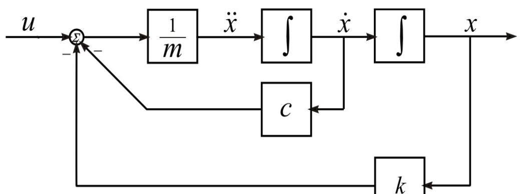

Fig. 3. *The diagram block of the system*

In a realistic system, parameters like mass, damping constant, and spring stiffness are unknown, but they can be varied in certain ranges: $m \in (1.8, 4.2)$, $c \in (0.8, 1.2)$ and $k \in (1.4, 2.6)$.

To control such a process, it is necessary to use a PI adaptive controller.

3.1. PI Adaptive Controller Designing

By selecting an adaptive law with three adjusting parameters, the downsides of the three unknown parameters process specifics, can help in finding of the proper values. To continue, the same adjusting mechanism that was detailed for the MIT rule is used further on. The mass - damper - spring process is a second order element with the following transfer function:

$$G(s)=\frac{Y(s)}{U(s)}=\frac{1/m}{s^2+(c/m)s+k/m}=\frac{\alpha_1}{s^2+\alpha_2 s+\alpha_3}. \quad (4)$$

For the reference model, a second order transfer function is selected:

$$G_m(s)=\frac{Y_m(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}. \quad (5)$$

A perfect following of the model reference is achieved with the PI control law [4], [8], [9]:

$$u(t)=k_1 r(t)-k_2 y(t)-k_3\dot{y}(t). \quad (6)$$

By inserting Equation (4) into Equation (6), the MIT rule is applied, where p is the differential operator:

$$Y(s)=G(s)U(s)=\frac{\alpha_1}{s^2+\alpha_2 s+\alpha_3}[k_1 R(s)-k_2 Y(s)-k_3 sY(s)]\Rightarrow$$
$$Y(s)=\frac{\alpha_1 k_1 R(s)}{s^2+(\alpha_2+\alpha_1 k_3)s+(\alpha_1 k_2+\alpha_3)}\Leftrightarrow$$
$$y(t)=\frac{\alpha_1 k_1 r(t)}{p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3)}. \quad (7)$$

As a consequence, the error is:

$$e(t)=\left(\frac{\alpha_1 k_1 r(t)}{p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3)}-\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right)r(t). \quad (8)$$

The sensitivity derivatives are obtained by taking the partial derivatives of the error and considering the controller parameters:

$$\frac{\partial e(t)}{\partial k_1}=\frac{\alpha_1 r(t)}{p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3)},$$
$$\frac{\partial e(t)}{\partial k_2}=\frac{-\alpha_1{}^2 k_1 r(t)}{(p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3))^2}=-\frac{\alpha_1 y(t)}{p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3)},$$
$$\frac{\partial e(t)}{\partial k_3}=\frac{-\alpha_1{}^2 p k_1 r(t)}{(p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3))^2}=-\frac{\alpha_1 \dot{y}(t)}{p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3)}. \quad (9)$$

Due to the fact that the process parameters are unknown, none of the above three equations can be used. The below approximation is required in order to overcome such an impediment:

$$p^2+(\alpha_2+\alpha_1 k_3)p+(\alpha_1 k_2+\alpha_3)=p^2+2\zeta\omega_n p+\omega_n^2. \quad (10)$$

In conclusion, the adjustment for the controller parameters is:

$$\frac{dk_1(t)}{dt}=-\gamma\left(\frac{1}{p^2+2\zeta\omega_n p+\omega_n^2}r(t)\right)e(t),$$
$$\frac{dk_2(t)}{dt}=\gamma\left(\frac{1}{p^2+2\zeta\omega_n p+\omega_n^2}y(t)\right)e(t),$$
$$\frac{dk_3(t)}{dt}=\gamma\left(\frac{1}{p^2+2\zeta\omega_n p+\omega_n^2}\dot{y}(t)\right)e(t). \quad (11)$$

where parameter α_1 is introduced in the adaptation gain γ.

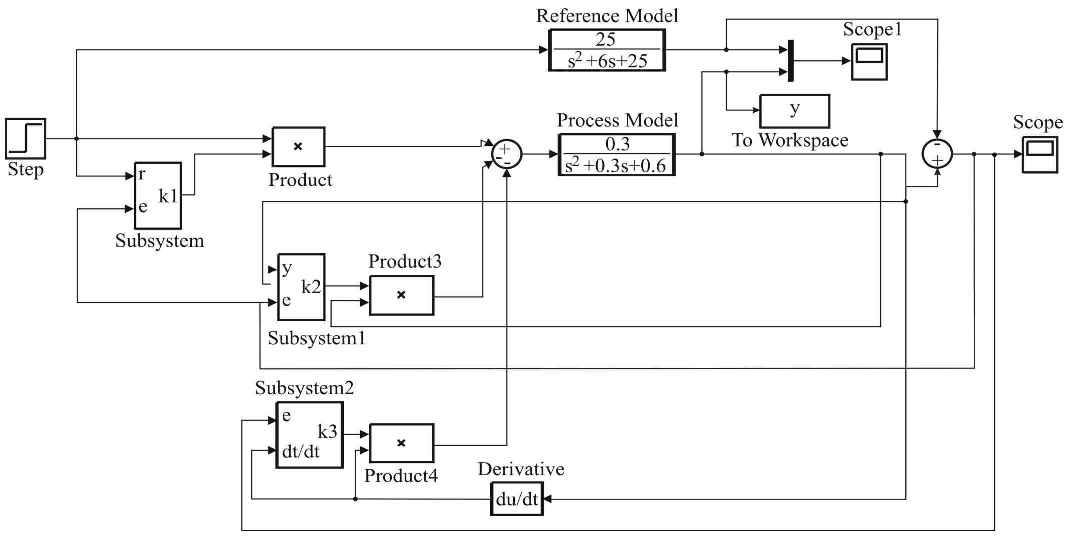

Fig. 4. *The simulation scheme*

4. Simulation Results

Both, the PI controller and the process have been designed in continuous time domain. The closed loop control system was simulated in Matlab/Simulink for the nominal values of the uncertain parameters (Figure 4) [10].

For the process model the authors proposed three scenarios: the first one is using the nominal values of the parameters ($y_1(t)$), the second is using the minimum values of the parameters ($y_2(t)$), and the third is using the maximum values of the parameters ($y_3(t)$).

By imposing a value equal to 3.5 for adaption gain and by analysing the signals' progression ($y_1(t)$, $y_2(t)$, $y_3(t)$), it can be deducted that system is adjustable. Moreover, in Figure 5 (where for a period of 100 seconds, a step input was applied), by analysing the signals it can be concluded that the system's performances are acceptable.

5. Conclusions

The current study presents in a rigorous way the experiments performed by the authors and pertaining to an adaptive controller designed for a mass - damper - spring process.

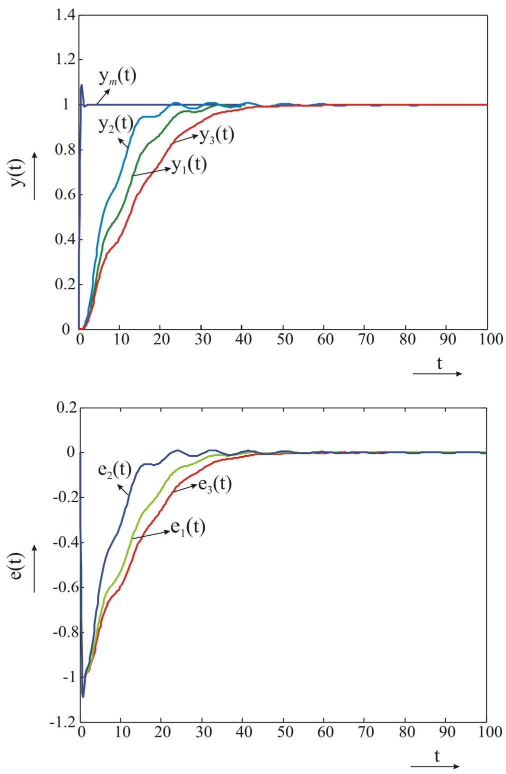

Fig. 5. *Progression of the output signals ($y(t)$, $y_m(t)$) and adjustment error signal ($e(t)$)*

The best experimental results have been obtained by applying a PI adaptive controller and only for a single scenario - when the parameters' nominal values have been set.

The MIT rule provides satisfactory results but does not guarantee coverage or stability.

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