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SLIDING MODE CONTROLLER DESIGN FOR ROBOT MANIPULATORS

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Abstract: *This paper deals with the design of sliding mode controller for a robot manipulator. Due to its order reduction property and its low sensitivity to disturbances and plant parameter variations, sliding mode control is an efficient tool to control complex high-order dynamic plants. The approach is based on a method called the reaching law method, which influences the dynamic quality of the system during the reaching phase, and providing the means for controlling the chattering level. The control scheme is validated through a set of simulations on a Matlab/Simulink package, and the simulation results confirmed the theoretical conclusions.*

Key words: *variable structure systems, sliding mode control, reaching law method, robot manipulators.*

1. Introduction

The focus of much of the research in the area of control systems theory during the seventies and eighties has addressed the issue of *robustness* - i.e. designing controllers with the ability to maintain stability and performance in presence of discrepancies between the plant and model. One nonlinear approach to robust controller design which emerged during this period is the variable structure control (VSC) systems methodology [9], [10].

VSC concepts have been utilised in the design of robust regulators, model-reference systems, adaptive schemes, tracking systems and state observers. The ideas have successfully been applied to problems as diverse as control of electrical motors, automatic flight control, mechanical systems, chemical processes, control of robot manipulators, observers and signal reconstruction [2], [4], [11].

The term *sliding mode control* (SMC) first appeared in the context of variable structure systems. Sliding modes became the principal operational mode for this class of control systems. Practically all design methods for variable structure systems are based on deliberate introduction of sliding modes. Due to its order reduction property and its low sensitivity to disturbances and plant parameter variations, SMC is an efficient tool to control complex high-order dynamic plants operating under uncertainty conditions which are common for many processes of modern technology.

The major drawback of SMC is the so-called *chattering phenomenon*. Such a phenomenon consists of the oscillation of the control signal, tied to the discontinuous nature of the control strategy, at a frequency and with an amplitude capable of disrupting, damaging or, at least, wearing the controlled physical system.

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The robustness of SMC is strictly connected to the high-frequency oscillations of the control signal. Yet, in practical applications the oscillation frequency and amplitude are obviously finite so that a degradation of the performances occurs. Two main causes have been identified. First, fast dynamics in the control loop which were neglected in the system model, are often excited by the fast switching of sliding mode controllers. Second, digital implementations in microcontrollers with fixed sampling rates may lead to discretization chatter. Several solutions have been proposed in the research literature to eliminate or reduce the chattering [8], [11].

Robots are complex mechanical systems with highly nonlinear dynamics. Hence high-performance operation requires nonlinear control designs to fully exploit a robot's capabilities [6-8].

This paper is organized as follows. Section II describes the dynamic model of robot manipulator. Section III mainly presents the principle of sliding mode and control algorithm. In Section IV, application of the above results to a robot manipulator is illustrated.

2. Mathematical Model of Robot Manipulator

A large number of control problems for mechanical systems are based on controlling the position or location of a mass using a force or a torque as the input variable. The typical example is a robotic arm or robot manipulator with n links connected by n joints with input forces/torques which are the outputs of actuators, often electrical actuators, with their own complex dynamics. These actuator dynamics are usually neglected (in the first step) in control design for the electromechanical system, assuming they are stable and considerably faster than the inertial dynamics of the masses. Also, other dynamics such as structural flexibilities are often neglected when deriving a basic model for the mechanical system. In practice this leads to the chattering problem.

For a large class of holonomic robot systems a continuous-time dynamic model can be written in configuration space as [1], [5]:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{F}_v(\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) = \tau\,, \quad (1)$$

where $\boldsymbol{q} \in R^{nx1}$ denotes the joint configuration variables (translational or rotational) of the n robot links; $\boldsymbol{M}(\boldsymbol{q}) \in R^{nxn}$ is the symmetric, positive definite inertia mass matrix; $\boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in R^{nxn}$ comprises Coriolis and centripetal forces; vector $\boldsymbol{F}_v(\dot{\boldsymbol{q}}) \in R^{nx1}$ describes viscous friction; vector $\boldsymbol{G}(\boldsymbol{q}) \in R^{nx1}$ contains the gravity terms; $\tau \in R^{nx1}$ is the control torque vector.

Define a $2n$-dimensional state vector $\boldsymbol{x}$ as:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}, \quad (2)$$

then the nonlinear plant in question (1) is described as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x}) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}\,, \quad (3)$$

where:

$$\boldsymbol{u} = \tau, \quad \boldsymbol{A}(\boldsymbol{x}) = $$
$$= \begin{bmatrix} \boldsymbol{x}_2 \\ -\boldsymbol{M}^{-1}(\boldsymbol{x}_1)[\boldsymbol{N}(\boldsymbol{x})\boldsymbol{x}_2 + \boldsymbol{F}_v(\boldsymbol{x}_2) + \boldsymbol{G}(\boldsymbol{x}_1)] \end{bmatrix},$$

$$\boldsymbol{B}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ \boldsymbol{M}^{-1}(\boldsymbol{x}_1) \end{bmatrix}.$$

This is the $2n^{\text{th}}$-order system having n input.

3. Design of Sliding Mode Controller for Robot Manipulator

VSC systems comprise a collection of different, usually quite simple, feedback control laws and a decision rule. Depending on the status of the system, a decision rule, often termed the *switching function*, determines which of the control laws is "on-line" at any one time. The transient dynamics of a VSC system consists of two modes: a "reaching mode" (or "nonsliding mode"), followed by a "sliding mode". Therefore the design of VSC involves, first, the design of an appropriate n-dimensional switching function $\boldsymbol{s}(\boldsymbol{x})$ for a desired sliding mode dynamics, and second, the design of a control for the reaching mode such that a reaching condition is met. The desired sliding mode dynamics is usually a fast and stable error-free response void of overshoot (an asymptotic convergence to the final state will be achieved in sliding mode). For the reaching mode, the desired response usually is to reach the switching manifold, described by:

$$\boldsymbol{s}(\boldsymbol{x}) = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{x} = 0, \tag{4}$$

in finite time with small overshoot with respect to the switching manifold.

For an n-input system, there are n switching functions and $2^n - 1$ sliding manifolds of different dimensions. The first m of them are designated as:

$$S_i = \{\boldsymbol{x} | s_i = \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x} = 0\}, \quad i = 1, \dots, n, \tag{5}$$

which may be called basic sliding manifolds since each of them is associated with a single switching function.

The dynamical behaviour of the system when confined to the surface is described as the *ideal sliding motion*. The advantages of obtaining such a motion are twofold: firstly there is a reduction in order and secondly the sliding motion is insensitive to parameter variations implicit in the input channels.

Gao and Hung [3] proposed a new approach, based on a new method called the *reaching law method*, for the design of VSC of nonlinear systems.

The method simultaneously takes care of ensuring the reaching condition, arranging the logic for the free-order switching, influencing the dynamic quality of the system during the reaching phase, and providing the means for controlling the chattering level. The procedure of using this method is straightforward and easy to carry out, even for nonlinear systems.

The reaching law is a differential equation which specifies the dynamics of a switching function $\boldsymbol{s}(\boldsymbol{x})$. The differential equation of an asymptotically stable $\boldsymbol{s}(\boldsymbol{x})$ is itself a reaching condition. In addition, by the choice of the parameters in the differential equation, the dynamic quality of VSC system in the reaching mode can be controlled. A practical general form of the reaching law is:

$$\dot{\boldsymbol{s}} = -\boldsymbol{Q}\,\mathbf{sgn}(\boldsymbol{s}) - \boldsymbol{K}\boldsymbol{h}(\boldsymbol{s}), \tag{6}$$

where:

$$\begin{aligned}
&\boldsymbol{Q} = \mathrm{diag}[q_1, \dots, q_n],\ q_i > 0,\\
&\mathbf{sgn}(\boldsymbol{s}) = [\mathrm{sgn}(s_1), \dots, \mathrm{sgn}(s_n)]^{\mathrm{T}},\\
&\boldsymbol{K} = \mathrm{diag}[k_1, \dots, k_n],\ k_i > 0,\\
&\boldsymbol{h}(\boldsymbol{s}) = [h_1(s_1), \dots, h_n(s_n)]^{\mathrm{T}},\\
&s_i h_i(s_i) > 0,\ h_i(0) = 0.
\end{aligned}$$

Tree practical special cases of (6) are given below.

1) Constant rate reaching:

$$\dot{\boldsymbol{s}} = -\boldsymbol{Q}\mathbf{sgn}(\boldsymbol{s})\,. \tag{7}$$

This law forces the switching variable $\boldsymbol{s}(\boldsymbol{x})$ to reach the switching manifolds S at a constant rate $|\dot{s}_i| = -q_i$. The merit of this reaching law is its simplicity. But, as will be shown later, if q_i is too small, the reaching time will be too long. On the other hand, a q_i too large will cause severe chattering.

2) Constant plus proportional rate reaching:

$$\dot{\boldsymbol{s}} = -\boldsymbol{Q}\mathbf{sgn}(\boldsymbol{s}) - \boldsymbol{K}\boldsymbol{s}\,. \tag{8}$$

Clearly, by adding the proportional rate term $-\boldsymbol{Ks}$, the state is forced to approach the switching manifolds faster when $\boldsymbol{s}$ is large. It can shown that the reaching time for $\boldsymbol{x}$ to move from an initial state $\boldsymbol{x}_0$ to the switching manifold S_i is finite, and is given by:

$$T_i = \frac{1}{k_i}\ln\frac{k_i|s_i| + q_i}{q_i}\,. \tag{9}$$

3) Power rate reaching:

$$\dot{s}_i = -k_i|s_i|^{\alpha}\,\mathrm{sgn}(s_i),\ 0 < \alpha < 1,\ i = 1,\ldots,n. \tag{10}$$

This reaching law increases the reaching speed when the state is far away from the switching manifold, but reduces the rate when the state is near the manifold. The result is a fast reaching and low chattering reaching mode. Integrating (10) from $s_i = s_{i0}$ to $s_i = 0$ yields:

$$T_i = \frac{|s_{i0}|^{1-\alpha}}{(1-\alpha)k_i}, \quad i = 1,\ldots,n, \tag{11}$$

showing that the reaching time T_i is finite. Thus power rate reaching law gives a finite reaching time. In addition, because of the absence of the $-\boldsymbol{Q}\mathbf{sgn}(\boldsymbol{s})$ term on the right-hand side of (10), this reaching law eliminates the chattering.

Having selected the reaching law equation, the control law can now by determined. Compute the time derivative of $\boldsymbol{s}(\boldsymbol{x})$ along the reaching mode trajectory, then from (4) and (6):

$$\begin{aligned}\dot{\boldsymbol{s}}(\boldsymbol{x}) &= \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{x}}\boldsymbol{A}(\boldsymbol{x}) + \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{x}}\boldsymbol{B}(\boldsymbol{x})\boldsymbol{u} = \\ &= -\boldsymbol{Q}\mathbf{sgn}(\boldsymbol{s}) - \boldsymbol{K}\boldsymbol{h}(\boldsymbol{s}).\end{aligned} \tag{12}$$

Noting that the matrix $(\partial \boldsymbol{s}/\partial \boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})$ is nonsingular, this equation is solved for the control law, giving:

$$\begin{aligned}\boldsymbol{u} = &-\left[\frac{\partial \boldsymbol{s}}{\partial \boldsymbol{x}}\boldsymbol{B}(\boldsymbol{x})\right]^{-1} \times \\ &\times\left[\frac{\partial \boldsymbol{s}}{\partial \boldsymbol{x}}\boldsymbol{A}(\boldsymbol{x}) + \boldsymbol{Q}\mathbf{sgn}(\boldsymbol{s}) + \boldsymbol{K}\boldsymbol{h}(\boldsymbol{s})\right].\end{aligned} \tag{13}$$

This control law appears independent of system perturbation and external disturbances, which is not realistic. In fact, the control $\boldsymbol{u}$ does depend on perturbation and disturbances, and it should include their parameters. A final but important note is that the control law (13), obtained via a reaching law, automatically leads to the free-order switching scheme. From the practical point of view, this scheme appears to be the most efficient.

The principle of designing SMC law for arbitrary-order plants is to force the error and derivative of error of a variable to zero. The robot arm is to track a desired motion $q^d(t)$. Define an error vector:

$$\boldsymbol{e} = \begin{bmatrix}\boldsymbol{e}_1 \\ \boldsymbol{e}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{q}^d - \boldsymbol{x}_1 \\ \dot{\boldsymbol{q}}^d - \boldsymbol{x}_2\end{bmatrix}, \tag{14}$$

and then define an n-dimensional vector switching function:

$$s(e) = Ce = [\Lambda \quad I]\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \Lambda\, e_1 + \dot{e}_1, \quad (15)$$

where $\dot{e}$ is the tracking speed error and:

$$\Lambda = \text{diag}\,[\lambda_1, \ldots, \lambda_n] \ , \lambda_i > 0,$$

that determine the bandwidth of the system.

Adopt the reaching law (8), and taking the time derivative of (15) gives:

$$\dot{s}(e) = \Lambda \dot{e}_1 + \dot{e}_2 = \Lambda \dot{e}_1 + \ddot{q}^d + M^{-1}(q) \times \\ \times [N(q,\dot{q})\dot{q} + F_v(\dot{q}) + G(q) - \tau], \quad (16)$$

and the final control law:

$$\tau = M(q)[Q\text{sgn}(s) + Ks + \Lambda \dot{e}_1 + \ddot{q}^d] + \\ + N(q,\dot{q})\dot{q} + F_v(\dot{q}) + G(q). \quad (17)$$

4. Design Example

Consider the two-link planar robot arm in Figure 1, for which the vector of generalized coordinates is $q = [\theta_1 \quad \theta_2]^{\mathrm{T}}$, θ_1 and θ_2 are absolute joint angles. Let a_1, a_2 be arm lengths, and l_1, l_2 the distances of the centres of mass of the two links from the respective joint axes. Also let m_{l1}, m_{l2} be the masses of the two links, and finally, I_{l1}, I_{l2} the moments of inertia relative to the centres of mass of the two links, respectively. The values of these parameters are given as: $a_1 = a_2 = 1\,\text{m}$; $l_1 = l_2 = 0.5\,\text{m}$; $m_{l1} = 20\,\text{kg}$, $m_{l2} = 10\,\text{kg}$; $I_{l1} = 0.8\,\text{kgm}^2$, $I_{l2} = 0.2\,\text{kgm}^2$.

The arm dynamics is described by (1), where the gravitational effect and viscous friction are neglected:

$$M(\theta)\ddot{\theta} + N(\theta,\dot{\theta})\dot{\theta} = \tau, \quad (18)$$

where:

$$M(\theta) = \begin{bmatrix} m_{11}(\theta_2) & m_{12}(\theta_2) \\ m_{21}(\theta_2) & m_{22} \end{bmatrix},$$

$$m_{11} = I_{l1} + m_{l1} l_1^2 + I_{l2} + \\ + m_{l2}(a_1^2 + l_2^2 + 2a_1 l_2 \cos\theta_2),$$

$$m_{12} = m_{21} = I_{l2} + m_{l2}(l_2^2 + a_1 l_2 \cos\theta_2),$$

$$m_{22} = I_{l2} + m_{l2} l_2^2,$$

and:

$$N(\theta,\dot{\theta})\dot{\theta} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \\ = \begin{bmatrix} n_{11}\dot{\theta}_1 + n_{12}\dot{\theta}_2 \\ n_{21}\dot{\theta}_1 \end{bmatrix},$$

$$n_{11} = -m_{l2} a_1 l_2 \dot{\theta}_2 \sin\theta_2,$$

$$n_{12} = -m_{l2} a_1 l_2 (\dot{\theta}_1 + \dot{\theta}_2)\sin\theta_2,$$

$$n_{21} = m_{l2} a_1 l_2 \dot{\theta}_1 \sin\theta_2.$$

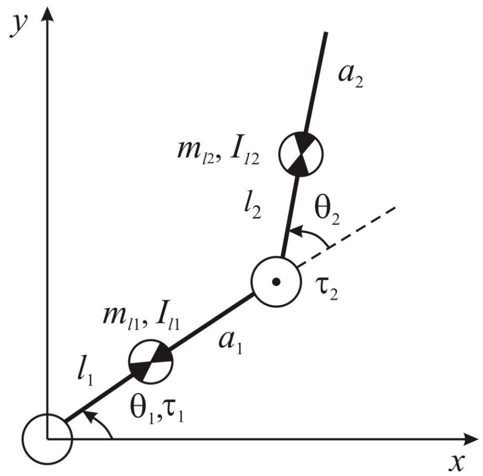

Fig. 1 *Two-link planar robot arm*

For the sliding mode controller technique, the sliding surface is chosen as:

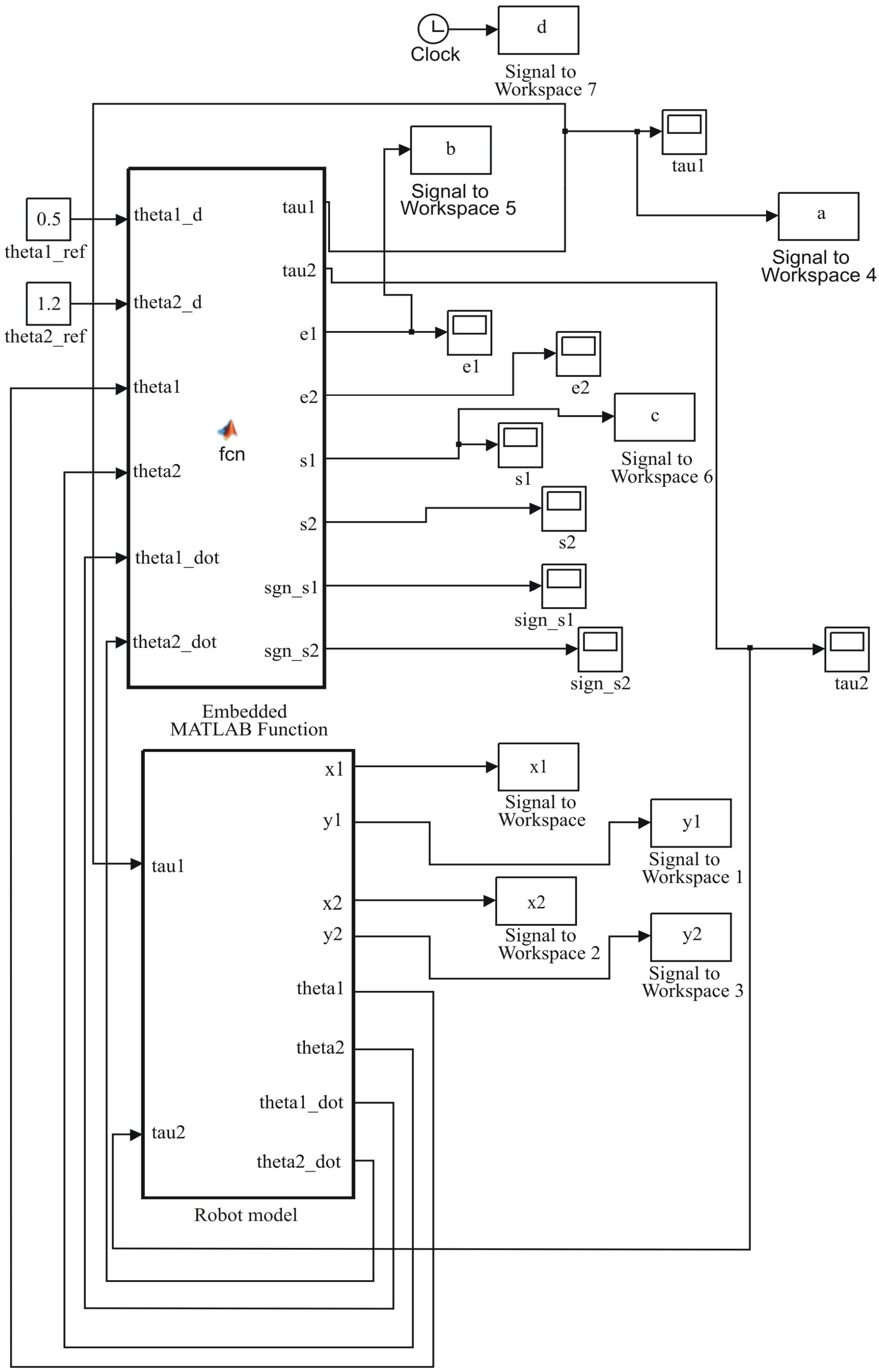

Fig. 1. *The simulation scheme*

$$s_i = \dot{e}_i + \lambda_i e_i, \quad \lambda_i > 0, \quad i = 1, 2,$$

where e_i define joint angle errors:

$$e_i = \theta_i^d - \theta_i, \quad i = 1, 2,$$

and θ_i^d is the desired position of the *i*th joint angle. Selecting the reaching law (8), resulted the switching law equations:

$$\dot{s}_i = -q_i \mathrm{sgn}(s_i) - k_i s_i, \quad i = 1, 2.$$

From (17), the VSC is:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\theta})[\boldsymbol{Q}\mathbf{sgn}(\boldsymbol{s}) + \boldsymbol{K}\boldsymbol{s} + \boldsymbol{\Lambda}\dot{\boldsymbol{e}} + \ddot{\boldsymbol{\theta}}^d] + \\ + \boldsymbol{N}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}, \tag{19}$$

where:

$$\boldsymbol{\Lambda} = \mathrm{diag}\,[\lambda_1, \lambda_2] = \mathrm{diag}\,[3.6, 4.5]\,.$$

To demonstrate the performance of the proposed control scheme, a set of computer simulation runs is carried out on an robot manipulator model.

The SMC system was simulated in Matlab/Simulink (Figure 1). Simulation results are shown in Figure 2 which contains the responses of errors e_1 and e_2 (in radians) of the joint angles θ_1 and θ_2, respectively, switching functions s_1 and s_2 (in radians), and control torques τ_1 and τ_2 (in Newton - meters). All four figures have the same time scale in seconds.

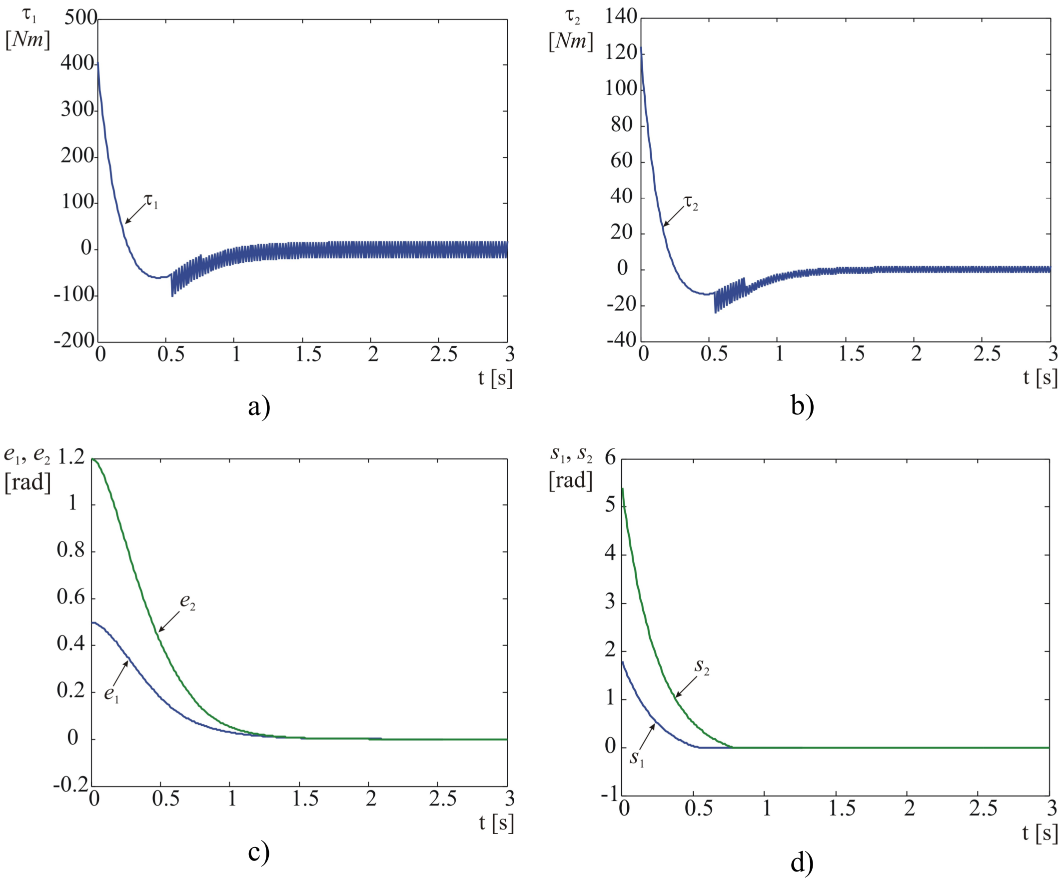

Fig. 2. *VSC of robot arm:* (*a*), (*b*) *controls* τ_1 *and* τ_2; (*c*) *joint angle errors* e_1 *and* e_2; (*d*) *reaching transients* s_1 *and* s_2 ($q_1 = q_2 = 6$, $k_1 = k_2 = 15$)

5. Conclusions

The paper presents the main steps to be followed in the design process of a sliding mode controller for robot manipulator. Even if the analytical approach is not completely detailed, it contains the major elements which need to be discussed. The performance of a control system depends on the types of controllers used. It was found that sliding mode controller gave very good performance for robot manipulator control. The robot manipulator model equations provided the basic for the design.

The effect of proposed method has been proven by simulations results. It is concluded that the proposed control topology produces better results for both dynamic and steady state operation.

Further research includes the study of influence of system perturbations and external disturbances and the experimental validation of the controller in a real plant.

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