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CONSIDERATIONS REGARDING THE OPTIMUM DESIGN OF PRESTRESSED ELEMENTS

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Abstract: *Engineering education in our universities tackles the theory of prestressed elements as part of the studies of reinforced concrete. However, the prestressing problem has a much more general nature. It is known that in the case of nonhomogenous elements, during the elastic-linear behaviour, it is not possible to reach the integral strength capacity in each material. This disadvantage may be removed by prestressing the material with higher strength. Thus, we obtain prestressed steel elements or another combination of two different materials. This paper presents the general theory of the prestressed elements, using the equivalent section method.*

Key words: *engineering education, prestressed elements, nonhomogeneous elements, strength capacity, high strength.*

1. Introduction

It is known that by prestressing structures we can obtain an optimum distribution of the stresses and strains. The result of this process is an economic design for strength elements. Although this principle has been known for a very long time, an engineering calculus of these structures has been developed only recently.

The prestressing principle is shown in Figure 1 where it is very easy to observe that by the introduction of the initial stress, the strength capacity of the prestressed element is higher than the strength capacity of the nonprestressed element.

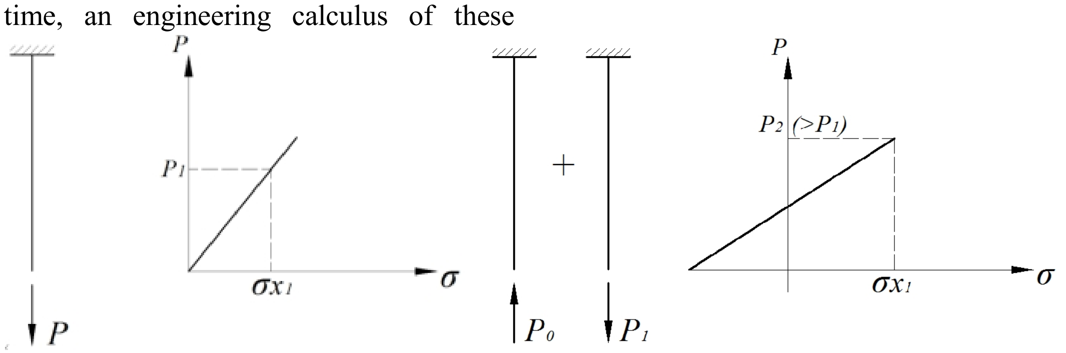

Fig. 1. *Prestressing principle*

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The more the value of the introduced stresses grows, the higher this strength capacity becomes. This observation is limited by the stability condition of the element during prestressing [3]. This disadvantage may by removed by applying step by step prestressing (Figure 2), but, in this case, it is necessary to have permanent service loads.

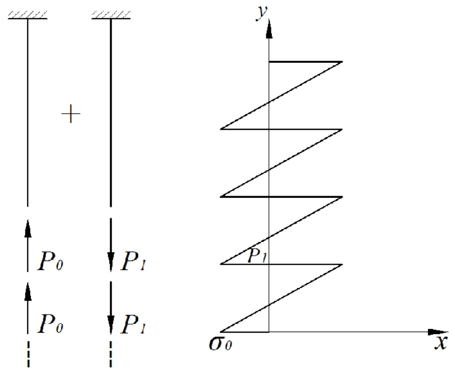


Fig. 2. *Prestressing step by step*

In our country the prestressing method was applied mainly for the reinforcement of old structures. For instance, at Saligny's bridge over the Danube, by loading the cantilevers of the main span (Figure 3) we obtained the reverse deformation of the bridge. On this deformed position we reinforced the flanges of the trusses from the main span. Annuling the load, the supplementary reinforcement will work even under its own weight.

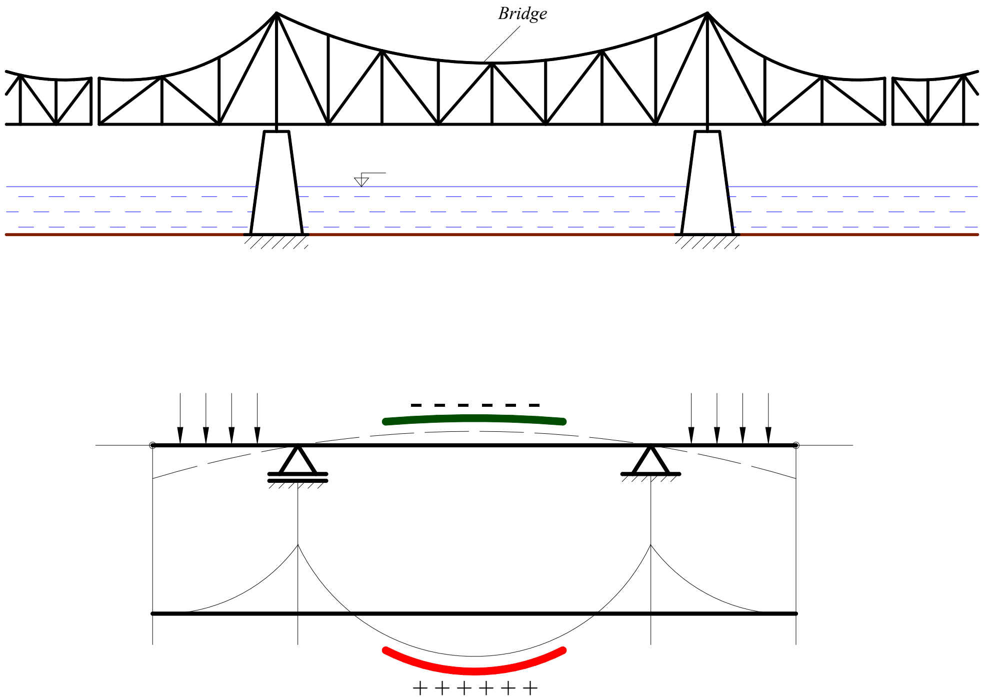


Fig. 3. *Saligny's bridge over the Danube river*

2. Prestressing System

The prestressing may be obtained using:
- tie bars or
- varied methods of execution.

Prestressing by Tie-Bars

Let us consider a beam having a tie-bar on its bottom face (Figure 4) [2].

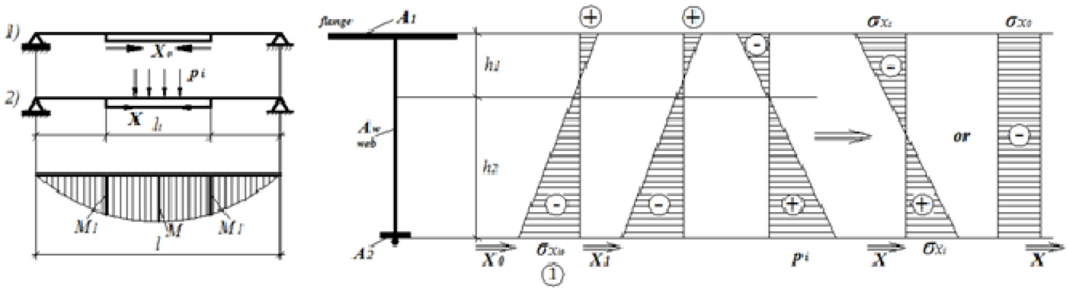


Fig. 4. *Beam with tie-bar*

1. We introduce effort X_0 in the tie bar. On the cross section we have the diagram 1.

$$\sigma_{xi_0} = -\frac{X_0}{A} - \frac{X_0 h_2}{W_2}$$

2. On this state we apply the external forces. The stresses are:

$$\sigma_{xi} = -\frac{X_0}{A} - \frac{X_0 h_2}{W_2} - \frac{X_1}{A} - \frac{X_1 h_2}{W_2} + \frac{M}{W_2} = -(X_0 + X_1)\left(\frac{1}{A} + \frac{h_2}{W_2}\right) + \frac{M}{W_2}$$

$$\sigma_{xs} = -\frac{X_0}{A} - \frac{X_0 h_2}{W_1} - \frac{X_1}{A} + \frac{X_1 h_2}{W_2} - \frac{M}{W_1} = -(X_0 + X_1)\left(\frac{1}{A} - \frac{h_2}{W_1}\right) - \frac{M}{W_1}$$

$$\sigma_{xt} = \frac{X_0 + X_1}{A_t}$$

3. Optimization Condition

a) $\sigma_{xi} = -\sigma_{xs} \Rightarrow X = \dfrac{W_1 - W_2}{W_1 + W_2} A \sigma_{xi}$

Remark: $W_1 > W_2$
that is: $h_2 > h_1$ (tie-bar must be fixed on the farthest fibbers)

b) $\sigma_{xi} = \sigma_{xs} \Rightarrow X = \dfrac{M}{h_2}$

homogeneous stress state

tie-bar carry out all the tensions

the cross section of the beam is subjected to compression. (but pay attention to its stability)

Strength conditions

- at the prestressing the tie-bars:

$$\left|\sigma_{xi_0}\right| \le \left|R_0\right|$$

- at the service state:

$$\sigma_{xi} \le R, \left|\sigma_{xs}\right| \le R, \sigma_{xt} \le R_t$$

and at the tie-bar ends:

$$\frac{M_1}{W_1} \le R$$

4. The Equivalent Section Method

Let us consider the prestressed beam, having any cross section (Figure 5). The stresses diagrams are:

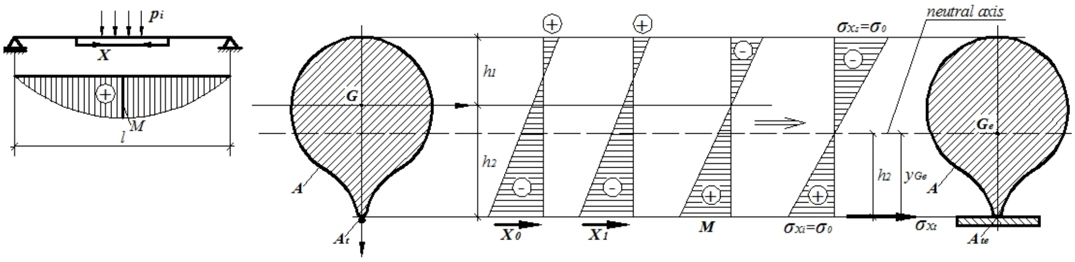


Fig. 5. *Prestressed beam. Diagrams*

We define a section which has:

- the area equal with actual area of the beam A;
- instead of tie bar, we have an area A_{te}.

$$A_{te} = \alpha A_t, \alpha = \frac{\sigma_{xt}}{\sigma_{xi}}$$

$$A_t = \frac{X}{\sigma_{xt}} = \frac{W_1 - W_2}{W_1 + W_2} A \frac{\sigma_{xi}}{\sigma_{xt}}$$

It is named a equivalent section to actual section of the beam.

Remark

The centroid of this equivalent section is:

$$y_{Ge} = \frac{Ah_2}{A + A_{te}} = \frac{Ah_2}{A + \alpha A_t} = \frac{Ah_2}{A + \alpha \dfrac{W_1 - W_2}{W_1 + W_2} \dfrac{A}{\alpha}} = h_2 \frac{W_1 + W_2}{2W_1}$$

$$y_{Ge} = h_2 \frac{\dfrac{I}{h_1} + \dfrac{I}{h_2}}{2\dfrac{I}{h_1}} = h_2 \frac{h_1 + h_2}{2h_1 h_2} h_1 = \frac{h}{2}$$

Ge is situated on the neutral axis of the diagram, which means that the equivalent section is subjected to plane bending [1].

So, that, insted of the prestressed cross section we can consider the equivalent section which, being subjected to plane bending, the calculus of the actual beam may be substituted by the calculus at plane bending of the equivalent section (Figure 6).

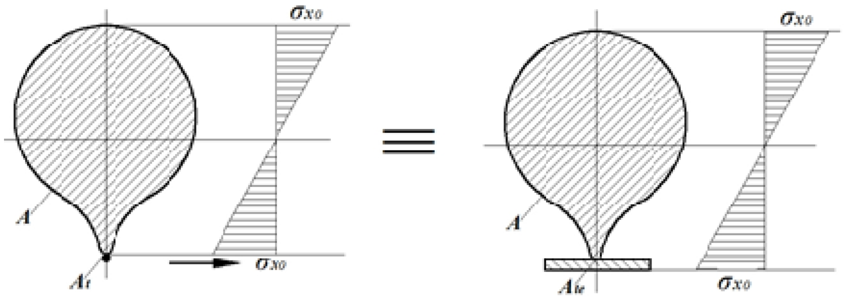


Fig. 6. *Equivalent section*

5. Calculus Stage

In function of σ_{xi}/σ_{xs} we determine neutral axis.

From condition that G_e to be on this axis it results A_{te}. We have the equivalent section [4].

By an usual bending computing we design the equivalent section:

$$Wz_{eq.\sec t} = \frac{M}{\sigma_{xi}}, Wz_{eq.\sec t} = Wz(t) \Rightarrow t$$

We determine the equivalent coefficient α, and results:

$$A_t = \frac{A_{te}}{\alpha}, A, (A = A_{eq} - A_{te})$$

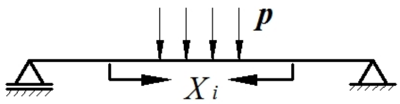
We determine the total effort of tie-bar:

$$X = \sigma_{xi} A_t$$

We verify the beam at the tie-bar prestressing:

- we find X_1 solving the system
- it results $X_0 = X - X_1$
- we verify the beam:

$$\sigma_{xi0} = -\frac{X_0}{A} - \frac{X_0 h_2}{W_2} \le \sigma_{0i}$$



and if:

- this condition is satisfied at limit (≤) nearby to   the calculus is stopped,
- contrary, we modify At and repeat the calculus.

We determine the length and the position of the tie-bar.

For double T sections the calculus is easier: ($\sigma_{xi} = \sigma_{xs}$)

- we consider an optimum equivalent section (Fig 7)

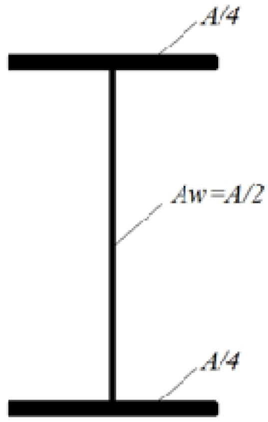


Fig. 7. *Optimum equivalent section*

and further, we follow the same calculation.

6. Conclusions

The paper presents a new and easy method for design of prestressed elements using the equivalent principle between the strength capacity of prestressed section and strength capacity of nonprestressed section. This method has two main advantages: a more rapid calculus and the possibility to obtain the optimum solution.

In this manner we eliminate the step by step calculus applying in the known method by which it is very difficult to arrive to an optimum solution in the design of these structures.

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