Bulletin of the *Transilvania* University of Braşov • Vol. 9 (58) No. 1 - 2016
Series I: Engineering Sciences

DYNAMIC MODELLING AND SIMULATION OF A PLANETARY SPEED INCREASER

Olimpiu MUNTEANU[1]

Abstract: *Many applications of wind turbines or hydro power plants include speed increasers meant to better harmonize the high input speed requirement of the electric generator with the typically low output speed of the converting turbine. This paper deals with a new variant of a planetary speed increaser used in wind or hydro turbines and presents some relevant aspects on the kinematic and dynamic modelling of the proposed planetary transmission. Based on a numerical example of a wind rotor - planetary speed increaser - DC electric generator, the dynamic response of this energy converting system is obtained by using the MATLAB Simulink software.*

Key words: *speed increaser, planetary transmission, dynamic, modelling, simulation.*

1. Introduction

The power output of wind turbines can be increased by applying several methods, such as enlarging the sweep area by increasing the length of blades or modifying the output speed of the wind rotor by changing the pitch angle of the blades [1, 8, 15, 17, 24].

Another known and largely applied method in wind turbine applications refers to the use of speed increasers, introduced between the wind rotor and the electric generator to increase the usually low wind rotor speed and thus to ensure better conditions for the electric generator operation. Typically, these speed increasers are gear transmissions with kinematic ratios between 5...30 for medium size turbines and up to 150 for large turbines [1, 2, 8]. The most common speed increasers use the fixed axes solution, especially for medium size turbines, or they are complex transmissions (combination of planetary gear units and transmissions with fixed axes) for larger ones [16].

The speed increasers with fixed axes have the drawback of large radial and axial size. The overall size can be significantly reduced by using planetary transmissions and thus better complying with the wind turbine design requirement to minimize system size.

Various planetary transmissions are addressed in literature, in the most of the cases used as speed reducers [5, 7], aiming at identifying their cinematic and dynamic features [2, 3, 22], or efficiency [4, 6, 18, 19, 23]. In describing the dynamic behaviour of a wind turbine, the wind speed has a major impact as this factor directly influences the wind rotor mechanical characteristic [20, 24].

[1] Renewable Energy System and Recycling Research Centre, *Transilvania* University of Braşov.

Based on these considerations, the paper presents the kinematic and dynamic modelling of a planetary increaser with two satellite gears serially mounted able to achieve higher kinematic ratio and decreased radial size. As numerical example, an application of a 0.5 kW wind turbine and a kinematic ratio of 10 is considered. Based on the identified transmission functions of this planetary increaser, the dynamic response of a wind turbine composed by wind rotor - speed increaser - DC electric generator is analysed using the MATLAB Simulink software and presented in the last part of the paper.

2. Modelling the Planetary Increaser

Optimal design of a planetary speed increaser requires to knowledge its structural, kinematic and dynamic features, identified by analytical modelling and numerical simulation.

2.1. Structural Aspects

The considered planetary speed increaser operates according to the block diagram in Figure 1 and has the structural scheme from Figure 2. The motion is transmitted from the input shaft (the carrier H), that supports two satellite gears mounted in series (2 and 3), through the fixed ring gear (4 = 0) toward the sun gear (1) attached to the output shaft. As a result, the wind rotor is fixed on the carrier H (low speed shaft), and the generator rotor is connected to the high speed shaft (1).

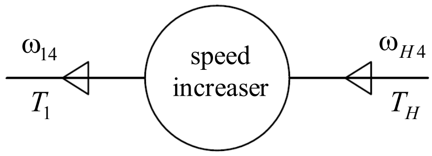


Fig. 1. *The block diagram of the planetary speed increaser*

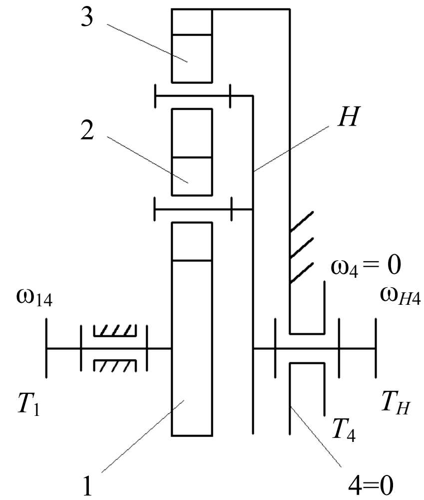


Fig. 2. *The structural scheme of the planetary speed increaser*

2.2. Kinematic Aspects

The speed transmitting function of the speed increaser from Figure 2 can be determined both graphically, based on the speed plan as depicted Figure 3, and analytically by applying the motion inversion in relation to the carrier H. The speeds are drawn in the construction plan, Figure 3, by applying the Kennedy's theorem of three instantaneous centres, obtaining the I_{20} centre as intersection of the lines CD and OA.

According to Figure 3, the angular speeds of gears and the kinematic ratio can be determined as follows:

- Input shaft:

$$v_H = OB \cdot \mathrm{tg}\,\delta_H = BC \cdot \mathrm{tg}\,\delta_3,$$

$$\omega_H = \frac{v_H}{r_H} = \frac{v_H}{OB} = \mathrm{tg}\,\delta_H. \tag{1}$$

- Output shaft:

$$v_E = OE \cdot \mathrm{tg}\,\delta_1,$$

$$\omega_1 = \frac{-v_E}{OE} = \frac{-v_E}{r_1} = -\mathrm{tg}\,\delta_1. \tag{2}$$

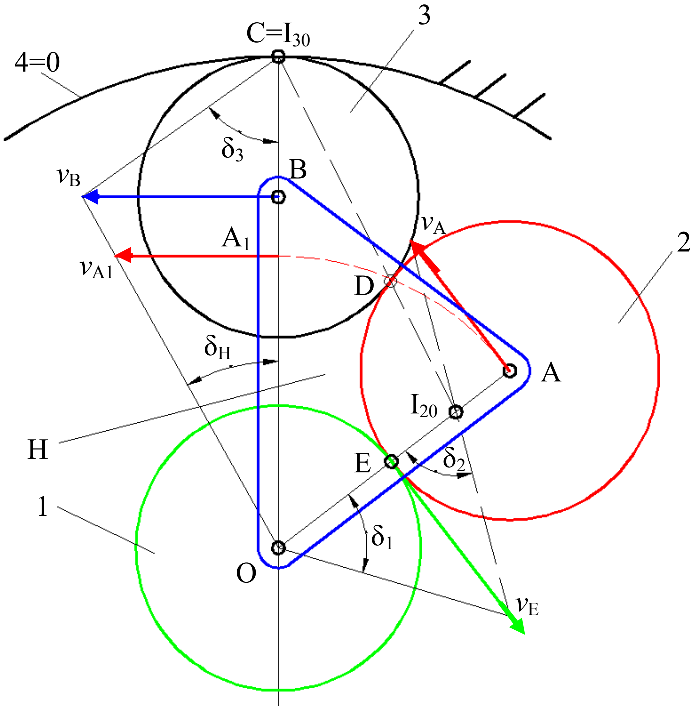


Fig. 3. *The speed plan of the 1 DOF planetary speed increaser*

- Amplification ratio:

$$i_a = \frac{\omega_1}{\omega_H} = -\frac{\mathrm{tg}\,\delta_1}{\mathrm{tg}\,\delta_H}. \quad (3)$$

The speed transmission function and the amplification ratio are analytically determined with the Relations (4) ... (7):

$$\omega_1 = \omega_H\left(1 - \frac{r_4}{r_1}\right) = \omega_H\left(1 - \frac{z_4}{z_1}\right). \quad (4)$$

By applying the motion inversion in relation to the carrier H [18], the fixed axes unit associated to the planetary transmission is obtained and as a result:

$$\omega_1 = \omega_H(1 - i_0), \quad (5)$$

$$i_{H1}^4 = \frac{\omega_H}{\omega_1} = \frac{-\omega_4}{\omega_1 - \omega_4} = \frac{1}{1 - i_0}, \quad (6)$$

$$i_a = i_{1H}^4 = \frac{\omega_1}{\omega_H} = 1 - i_0, \quad (7)$$

where $i_0 = z_4/z_1 > 1$ is the internal kinematic ratio of the planetary unit.

The amplification ratio i_a in relation with the internal kinematic ratio i_0 is represented in Figure 4.

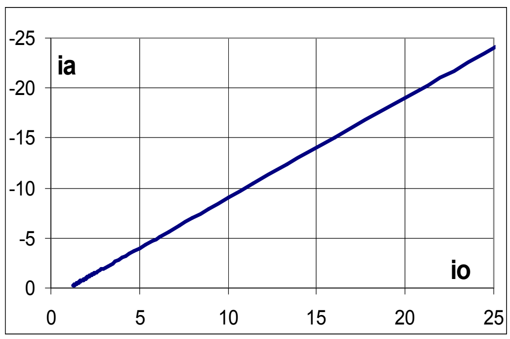


Fig. 4. *The amplification ratio variation vs. internal kinematic ratio*

2.3. Static Aspects

The efficiency of the speed increaser can be obtained using the Relation [18]:

$$\eta_a = \eta_{H1}^4 = \frac{-\omega_{14} T_1}{\omega_{H4} T_H} = \frac{-T_1/T_H}{\omega_H/\omega_1} = \frac{\overline{i_{H1}^4}}{i_{H1}^4},$$

where $\overline{i_{H1}^4}$ is the static ratio.

Introducing the internal efficiency $\eta_0 = \eta_{12}\eta_{23}\eta_{34}$ (η_{xy} is the efficiency of the *xy* gear with fixed axes) and according to the Relation (6):

$$\eta_a = \frac{1 - i_0}{1 - i_0 \eta_0^x}, \tag{8}$$

where:

$$x = \mathrm{sgn}(\omega_{1H} T_1) = \mathrm{sgn}\left(\frac{\omega_{1H} T_1}{-\omega_{14} T_1}\right) = \mathrm{sgn}\left(\frac{-i_0}{i_0 - 1}\right) = \mathrm{sgn}\left(\frac{i_0}{1 - i_0}\right) = -1,$$

where sgn is the signum function.

The variation of the speed increaser efficiency in relation with the internal kinematic ratio is plotted in Figure 5.

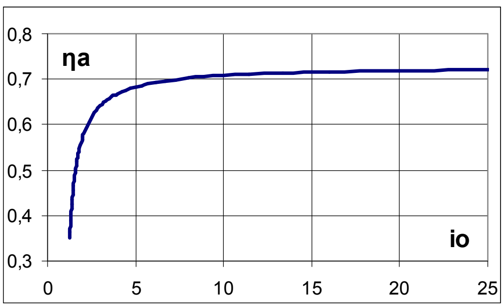


Fig. 5. *Variation of the planetary increaser efficiency vs. internal kinematic ratio*

Considering friction, the torque transmitting function is given by the following Equation:

$$T_1 = -i_{H1}^4 \eta_{H1}^4 T_H. \tag{9}$$

3. Dynamic Response

The modelling of a wind turbine dynamic response aims at establishing the time variation of angular speeds, angular accelerations and torques. The applied approach is based on the Newton-Euler method [9-14, 21], with the following assumptions:

- the transmission elements are rigid bodies made of steel;
- the mass centres of the gears are positioned on their axes of rotation;
- the mechanical moments of inertia of the outer shafts (Figure 6) incorporate also the moments of inertia of the mobile gears. For example, the momentum of inertia J_H cumulates the moments of inertia of the wind rotor, of the shaft H and of the satellite gears (2) and (3); respectively, J_1 includes the moments of inertia of the gear (5) and of the generator rotor;
- the effects of friction are considered by the speed increaser efficiency;
- the mechanical characteristics of the wind rotor and the generator of electric current are linear functions.

Accordingly, the dynamic model is obtained by solving the system of the wind turbine dynamic equations, formed by the speed increaser equations (Figure 6) and the mechanical characteristic of the wind rotor and of the electric generator:

- the planetary speed increaser Equations:

$$\begin{aligned} &\omega_1 = i_a \omega_H,\ \varepsilon_1 = i_a \varepsilon_H, \\ &T_1 + T_4 + T_H = 0,\ T_1 i_0 \eta_0^x + T_4 = 0, \\ &J_H \varepsilon_H = T_t - T_H, \\ &J_1 \varepsilon_1 = T_g - T_1, \\ &\omega_H T_H \eta_{H1}^4 + \omega_1 T_1 = 0; \end{aligned} \tag{10}$$

- mechanical characteristic of the wind rotor:

$$T_t = -a_t\omega_t + b_t\,; \tag{11}$$

• mechanical characteristic of the electric generator:

$$-T_g = a_g\omega_g - b_g\,. \tag{12}$$

Based on the Equalities $\omega_t = \omega_H$, $\omega_g = \omega_1$, $T_t = T_H$, $T_g = T_1$, by replacing the Relations (11) and (12) to Relation (10).

Solving the system of Equations (10), the dynamic equation of the wind turbine is obtained:

$$\varepsilon_1 A = -\omega_1 B + C\,, \tag{13}$$

where:

$$A = J_1 + \frac{J_H \eta_{H1}^4}{i_a^2}\,,$$

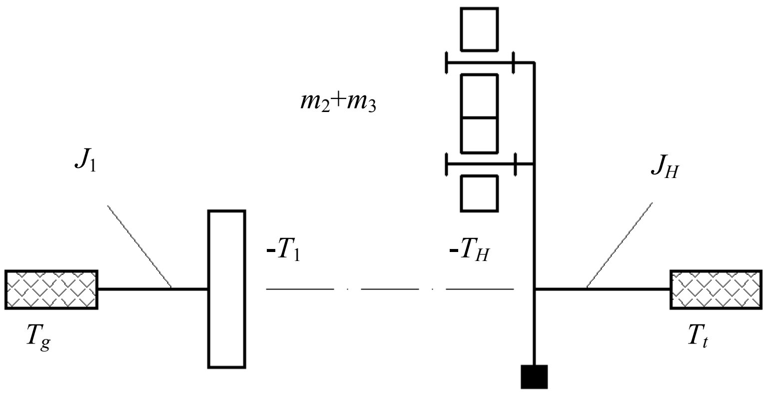
$$B = a_g + a_t\frac{\eta_{H1}^4}{i_a^2}\,,$$

$$C = b_g + b_t\frac{\eta_{H1}^4}{i_a}\,.$$

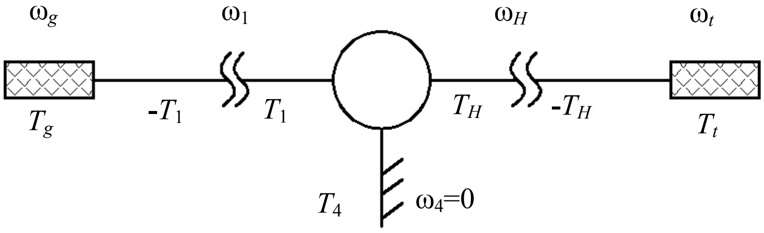
In the stady state (for $\varepsilon_1 = 0$), the equation for determining the angular speed of the electric generator rotor is obtained:

$$\omega_g = \frac{b_g + b_t\dfrac{\eta_{H1}^4}{i_a}}{a_g + a_t\dfrac{\eta_{H1}^4}{i_a^2}}\,. \tag{14}$$

The dynamic response of the wind turbine (wind rotor - speed increaser - electric generator), i.e. the time variation of the angular speeds, angular accelerations and torques is obtained using the MATLAB Simulink software, Figure 7.



a)



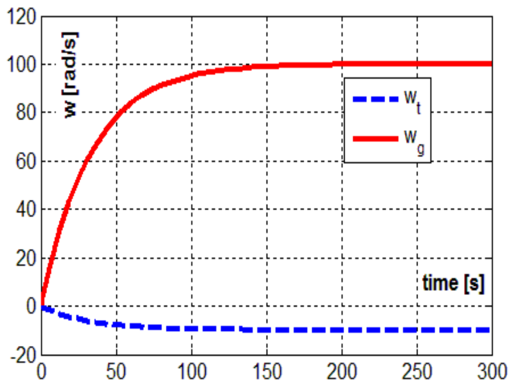
b)

Fig. 6. *Dynamic model of the planetary increaser: a) dynamic scheme*; *b) block diagram*

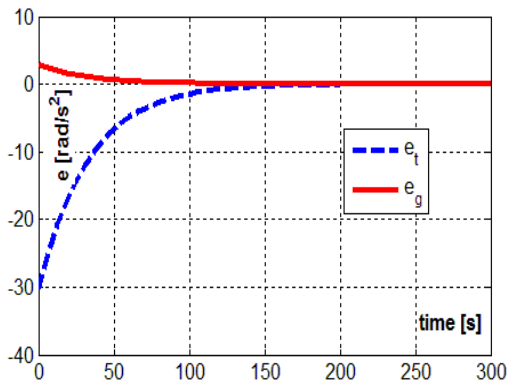
4. Results and Discussions

A numerical example is further considered based on the following values of the kinematic and dynamic parameters:

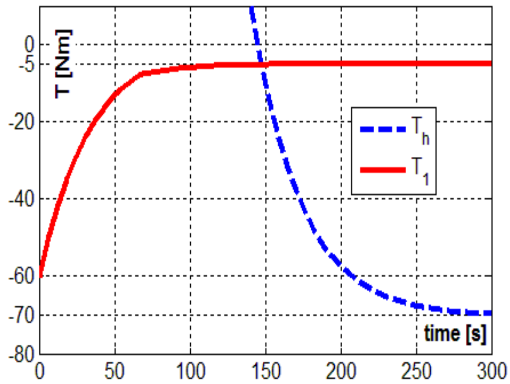
- internal kinematic ratio: $i_0 = 11$,
- amplification ratio: $i_a = -10$,
- internal efficiency: $\eta_0 = 0.729$,
- speed increaser efficiency: $\eta_a = 0.709$,
- mechanical moments of inertia: $J_H = 200$ kgm², $J_1 = 20$ kgm²,
- wind rotor constants: $a_t = 35.2229$ Nm/s, $b_t = -422.6748$ Nm,
- generator constants: $a_g = 0.4$ Nm/s, $b_g = 35$ Nm.



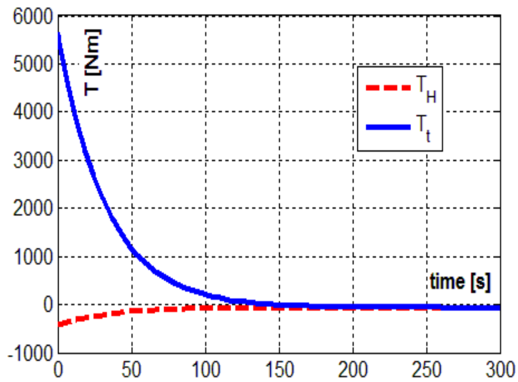
a)



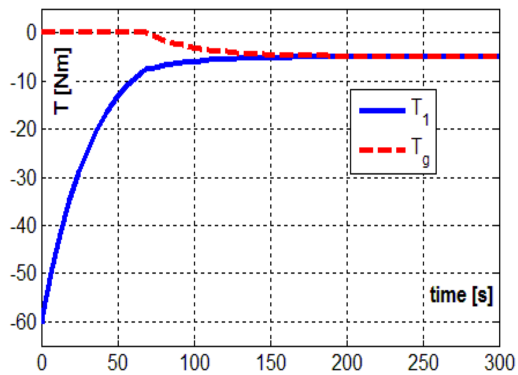
b)



c)



d)



e)

Fig. 7. *Numerical simulations: a) the angular speed, b) the angular acceleration, c) the torques, d) the torque of the input shaft, e) the torque of the output shaft*

Given the results shown in Figure 7, the analysed transmission goes in the steady state after approx. 200 s (Figure 7b), increases 10 times the input speed (Figure 7a) and reduces 14 times the torque (Figure 7c). For the considered moments of inertia, the electric generator is operating after approx. 70 s (Figure 7e).

In the working regime of the wind turbine (steady state), the torques of the wind rotor and of the input shaft (*H*) are equal, and similarly for the torques of the output shaft (1) and the generator rotor. The ratio between input and output torques depends on the amplification kinematic ratio and speed increaser efficiency.

The wind turbine operates in steady state with a wind rotor speed of 10 rad/s (95.5 rpm), and thus the electric generator rotor speed becomes 100 rad/s (955 rpm).

5. Conclusions

The paper presents the dynamic response of a wind turbine consisting of a wind rotor - planetary speed increaser - electric generator, obtained by using the MATLAB Simulink software, and based on the kinematic and dynamic modelling of the planetary speed increaser and mechanical characteristics of the wind rotor and of the electric generator.

The considered planetary transmission changes the direction of rotation (Figure 2), the input speed increases 10 times, while reducing 14 times the input torque (Figure 7) under the consideration of the gearing friction.

The considered wind turbine can produce power after approx. 70 s ($T_g \neq 0$, Figure 7e) and enters in the steady state after 200 s ($\varepsilon = 0$, Figure 7a).

The dynamic model may be useful in designing control systems for wind turbines whose control program can be established by considering certain conditions and operating parameters.

Other information may be obtained from the address: omunt@unitbv.ro.

References

1. Burton, T., Sharpe D., et al.: *Wind Energy Handbook*. John Wiley & Sons, LTD, West Sussex, 2001.
2. Climescu, O., Săulescu, R., et al.: *Specific Features of a Counter-Rotating Transmission for Renewable Energy Systems*. In: Environmental Engineering and Management Journal **10** (2011), p. 1105-1113.
3. Climescu, O., Jaliu, C., et al.: *Innovative Planetary Transmission Usable in Res*. In: Annals of the Oradea University. Fascicle of Management and Technological Engineering **XI** (2012), p. 2.11-2.16.
4. Climescu, O., Jaliu, C., et al.: *On the Efficiency of a Planetary Speed Increaser Usable in Small Hydros*. In: Power Transmissions. Mechanism and Machine Science **13** (2013), p. 259-268.
5. Cross, N.: *Engineering Design Methods*. J. Wiley & Sons, New York, 1994.
6. Diaconescu, D.V., Săulescu, R.: *On the Optimization the Efficiency for Simple Planetary Reducers*. In: The Ninth IFToMM International Symposium on Theory of Machines and Mechanisms, Bucharest, vol. II (2005), p. 335-340.
7. Dieter, G.: *Engineering Design*. McGraw Hill, Boston, 2000.
8. Hau, E.: *Wind Turbines: Fundamentals, Technologies, Application, Economics*. Springer, Germany, 2006.
9. Jaliu, C., Diaconescu, D., et al.: *Speed Multipliers Variants used in Wind Turbines and Hydro-Power Stations*. In: 11th International Research/Expert Conference Trends in the

Development of Machinery and Associated Technology, 2007, p. 1115-1118.

10. Jaliu, C., Diaconescu, D., et al.: *Kinematical and Dynamic Properties of a speed multiplier used in wind turbines*. In: Recent **8** (2007), p. 507-512.
11. Jaliu, C., Diaconescu, D., et al.: *Speed Multipliers for Renewable Energy Systems-Hydro and Wind*. In: International Conference on Renewable Energies and Power Quality, 2008, CD Proceedings.
12. Jaliu, C., et al.: *Dynamic Features of Speed Increasers from Mechatronic Wind and Hydro Systems*. In: Proceedings of EUCOMES'08, 2008, p. 355-373.
13. Jaliu, C., et al.: *Conversion Analysis of a Planetary Chain-Set Speed Reducer into a Speed Increaser to Be Used in RES*. In: Proceedings of the Third International Conference on Mechanical Engineering and Mechanics **1** and **2** (2009), p. 767-770.
14. Jaliu, C., et al.: *On the Dynamics of a Planetary Transmission for Renewable Energy Systems Plants*. In: WSEAS Transactions on Applied and Theoretical Mechanics **5** (2010) Issue 2, p. 146-155.
15. Johnson, G.: *Wind Energy Systems*. Manhattan, KS, 2006.
16. Leimann, D.: *Planetary Gear Unit for a Gearbox for a Wind Turbine*. Patent no. US20110077120 A1, 2011.
17. Manwell, J.F., Mcgowan, J.G., et al.: *Wind Energy Explained*. John Wiley & Sons, 2005.
18. Miloiu, G., Dudita, Fl., et al.: *Modern Mechanical Transmissions* (in Romanian). Bucureşti. Editura Tehnică, 1971.
19. Pahl, G., Beitz, W.: *Engineering Design*. Springer, London, 1995.
20. Patel, M.: *Wind and Solar Power Systems. Design, Analysis, and Operation*. Taylor & Francis Group, LLC, 2006.
21. Săulescu, R., et al.: *On a New Planetary Speed Increaser Drive Used in Small Hydros. Part II. Dynamic Model*. In: Proceedings of EUCOMES **5** (2010), p. 209-216.
22. Săulescu, R., Jaliu, C., et al.: *On the use of 2 DOF Planetary Gears as "Speed Increaser" in Small Hydros and Wind Turbines*. In: Proceedings of the ASME 2011, International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, IDETC/CIE, 2011, CD Proceedings.
23. Săulescu, R., Jaliu, C., et al.: *Differential Planetary Gear Transmission Usable in Renewable Energy Systems*. In: Proceedings of MeTrApp 2011. Mechanism and Machine Science, Vol. 3, Mechanisms, Transmissions and Applications, 2011, p. 275-282.
24. Tong, W.: *Wind Power Generation and Wind Turbine Design*. WIT Press, 2010.