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ABOUT THE CALCULUS OF THE PRESTRESSED BEAMS USING EQUIVALENT SECTION METHOD

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Abstract: *Due to the constant evolution of the consumer market, new building materials appear continuously. One such product is the new developed composite plaster mortar. The main purpose of the current research is to evaluate the adherence to the support layer of plaster mortars reinforced with polypropylene synthetic fibers. In this experimental study, sets of 2 samples of each recipe were materialized on the substrate of solid bricks. It is intended an analysis of the mechanical characteristics of the material, in order to indicate a specific amount of fibers in the matrix and to recommend some proper application areas.*

Key words: *polypropylene; synthetic fibers; adherence; plaster mortar; composite; reinforcement*

1. Introduction

A design method of the bent beams sections, prestressed by the high-strength tie-bar, is presented. The method leads to a relative simple calculus and it may be applied at the automatic design.[1],[3]
The method is based on the equivalent section principle (3,1971), which suppose that a prestressed section may be substituted, into calculus, by a nonprestressed section having the same strength capacity. According to this principle, the design of the prestressed beams supposes the following successive calculus stages:
- it is designed, at the total bending moment M, the beam section considering that it is an habitual, nonprestressed section (subjected to plane bending). This section is named equivalent section of the prestressed beam and it has the same strength capacity as well as the prestressed section;
- the tie-bar section is adapt (by the design experience). The equivalent area of the tie-bar (obtained by the multiplication of the actual area with the ratio between the tie-bar resistance and beam resistance measured at the level of the tie-bar) is substract from the section tensioned flange determined at the previous point. This new section and the tie-bar is the solution of the problemif the strength condition at the prestressing of the tie-bar is satisfied. Contrarly, a new section of the tie-bar is adopted and the design calculus is repeted.

The method supposes that a tie-bar list must be, initialy, fixed. From this list, by

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shown calculus, the initial tie-bar is chosen.

Though the solving principle has a general nature, being applied to the beams, having any shape of their sections, the calculus relations given in this paper refer to the double T sections or sections that can be assimilated with this shape.

Choosing the equivalent section is done by the optimum shape criteria (3, 1971), which determines how much from the total area of the section is distributed in the flanges so that the strength modulus can be maximal. Equivalent section method has the advantage that allows to determine the optimal section of prestressed beam.[2]

The pretension stress of the tie-bar amplifies entirely, in one stage, before the working load is applied. The tie-bar is settled at the bottom flange of the beam, which means it's strained area, and it's considered to be rectilinear, parallel with the element's flange.

The beam and the tie-bar can be made from any material. In the following, the beam-tie bar assembly is subjected in the linear elastic domain of behavior, both in the utilization phase and the prestress phase of the tie-bar (with X_0 effort).

The presented calculation procedure offers the possibility of differentiating the calculus strengthes at the two flanges of the beam (either because of the material that has different streght at tension and compression or because of using different materials in the two flanges), the differentiation being possible both in the utilization phase and the prestress phase of the tie-bar.[4]

2. Relations Calculation

The equivalent double T section (fig. 1a) is designed with the relations:

$$A_1 = \frac{1}{r\left(r+1\right)}\sqrt[3]{\frac{9M^2\left(r+1\right)}{8\text{a}R_i^2}}$$
$$A_{2e} = \frac{r}{r+1}\sqrt[3]{\frac{9M^2\left(r+1\right)}{8\text{a}R_i^2}} \qquad (1)$$
$$A_i = \frac{2}{r+1}\sqrt[3]{\frac{9M^2\left(r+1\right)}{8\text{a}R_i^2}}$$

Were:
r - represents the ratio between the the compression and tension strength of the steel beam;

$$r = \frac{R_c}{R_i} \qquad (2)$$

λ - web's ratio of slenderness, defined:

$$\text{a} = \frac{h_i}{t_i} \qquad (3)$$

M - total bending moment momentul used to design the beam.
The coefficient λ is set from local stability conditions of the beam's web. Knowing the area of the web A_i, can be calculated:

- thickness of teh web

$$t_i = \sqrt{\frac{A_i}{\text{a}}} \qquad (4)$$

- elevation of the web

$$h_i = \text{a} \cdot t_i \qquad (5)$$

The position of the section's center of gravity determined by (1) result from the condition:
$r = \frac{R_c}{R_i} = \frac{h_1}{h_2}$ which associated to the relation

$$h_1 + h_2 = h_i$$

leads to: $h_2 = \frac{h_i}{r+1}$ (6)

And

$$h_1 = rh_2 \qquad (7)$$

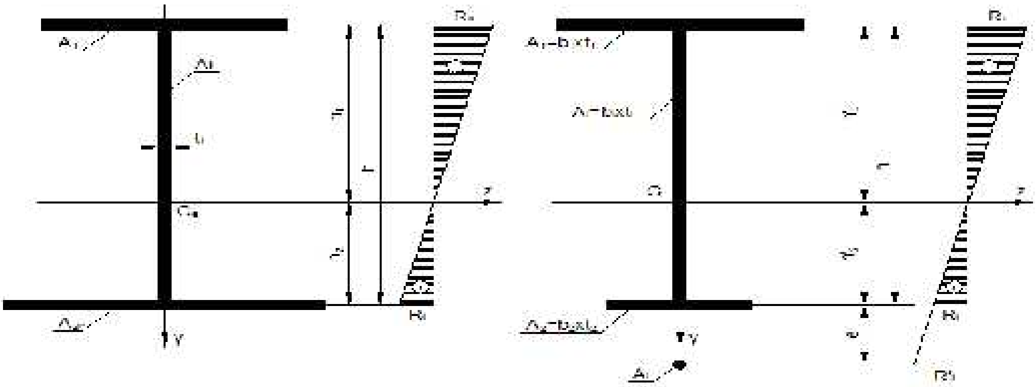

Fig. 1. *Double T section* a) *equivalent double T section* b) *prestressed section*

Knowing the tie-bar's effective area A_t is determined it's equivalent area:

$$A_{te} = nA_t$$

where the equivalence coefficient n represents the ratio between the strength of the tie-bar and the strength of the beam's steel measured at the level of the tie-bar:

$$n = \frac{R_t}{R_i^{'}} = \frac{R_t}{R_i} \cdot \frac{h_2}{h_2 + e} \tag{9}$$

If the equivalent area of the tie-bar is substracted from the strained flenge's area A_{2e}, is obtained:

$$A_2 = A_{2e} - A_{te} \tag{10}$$

The section determined by A_1, A_i and A_2 is the section of the prestressed beam and A_t is the area of the appropriate tie-bar. The dimensions of the flanges and the web result:

$$b_2 = \frac{A_2}{t_2}\ ;\ b_1 = \frac{A_1}{t_1}\ ;\ h_i = \frac{A_i}{t_i} \tag{11}$$

with the condition that the fallowing inequalities are satisfied too:

$$\frac{b_1}{t_1} \le m_1\ ;\ \frac{b_2}{t_2} \le m_2 \tag{12}$$

where in the dimensions t_1, t_2 are chosen from a list composed previously by the engineer, depending on the choises that are available or other limiting conditions that occur. The parameters m_1, m_2 and λ are set depending on the slenderness that the element must have.

In this way are obtined the area of the tie-bar and all of the cross section's dimensions which respect the strength condition, the standard type and the requested slenderness for each flange and web in part.

In the following it's still necessary to verify the beam at the prestressing of the tie-bar. For this purpose it is determined, initially, the geometrical characteristics of the cross section (fig. 1b):

- position of the center of gravity G:

$$h_1^{'} = \frac{A_1 \cdot \frac{t_1}{2} + A_i\left(t_1 + \frac{h_i}{2}\right) + A_2\left(h - \frac{t_2}{2}\right)}{A_i + A_1 + A_2}$$

$$h_2^{'} = h - h_1^{'}$$

Where:

$$h = h_i + t_1 + t_2$$

- moment of inertia with respect to z axis, strength module and the area of the prestressed section:

$$I_z = \frac{b_1 \cdot t_1^3}{12} + A_1\left(h_1^{'} - \frac{t_1}{2}\right)^2 - \frac{t_i \cdot h_i^3}{12} + A_i\left(\frac{h_i}{2} + t_1 - h_1^{'}\right)^2 + \\ + \frac{b_2 \cdot t_2^3}{12} + A_2\left(h_2^{'} - \frac{t_2}{2}\right)^2$$

$$W_{zs} = \frac{I_z}{h_1^{'}};\quad W_{zi} = \frac{I_z}{h_2^{'}};\quad A = b_1 t_1 + h_i t_i + b_2 t_2$$

The statically indeterminate system subjected to service loads is solved and it is obtained the value of the autotensioning stress X_1. Autotensioning stress of the tie-bar results:

$$X_0 = A_t R_t - X_1 \qquad (13)$$

The condition for prestressing that needs to be fulfilled is:

$$\left(R_0 - \pounds\right) \le \pm_{i0}\left| - \frac{X_0}{A} - \frac{X_0\left(h_2^{'} + e\right)}{W_{zi}} \le \left(R_0 + \pounds\right)\right. \qquad (14)$$

where R_0 is the calculus stress at the compressed face of the beam determined takeing account of the strength condition and the stability condition and ε is the admitted error from this strength.

If (14) is fulfilled, the cross section's design is finished. If not, are possible two alternatives:

a) if

$$\pm_{i0} << R_0 - \pounds \qquad (15)$$

the prestressing effort can be increased and therefore, from the list with tie-bars, it is chosen a superior one and the calculus is repeated.

b) if

$$\pm_{i0} > R_0 + \pounds \qquad (15')$$

the chosen tie-bar is too big and it will be chosed another one, and the the calculus is repeated.

In the final phase, as a additional verification, are determined the stresses from the beam's cross section extreme fibers, in the service phase:

$$\pm_i = \frac{-X_0 + X_1}{A} + \frac{M - \left(X_0 + X_1\right)\left(h_2^{'} + e\right)}{W_{zi}} \le R_i \qquad (16)$$

$$\pm_c = \left|\frac{-X_0 + X_1}{A} - \frac{M - \left(X_0 + X_1\right)\left(h_2^{'} + e\right)}{W_{zs}}\right| \le R_c \qquad (17)$$

as well as the maximum stress in the tie-bar:

$$\pm_t = \frac{X_0 + X_1}{A_t} \le R_t \qquad (18)$$

The method has a large field of application. It may be used at the design of the prestressed beams achieved from steel, reinforced concrete, wood, composite materials and any others materials which respect the assumptions of the linear-elastic calculus.

The number of the repeated stages of calculus depends on the experience of the designer (design-man) and for a normal experience these stages may be 2 max 3, but , however the calculus volume is much more reduced than in the step by step method , which is used today, still.

3. Conclusions

The equivalent section method is a general and direct method for the design of the bent beams achieved from the different materials which respect the linear-elastic assumptions .

The calculus may be made by hand and by automatical way too.

References

1. Mateescu, D., *Special steel constructions*, Technical Editure, 1962.
2. Juncan, N., *Contributions to the study of prestressed steel trusses,* doctoral dissertation, Cluj-Napoca 1969.
3. Precupanu, D., *Fast method regarding the calculus of poststressed steel beams*, I.P.I. Bulletin, fascicle 3-4, 1971.
4. Precupanu, D. *Optimum shape of double T section,* I.P.I. Bulletin, fascicle 3-4, 1973.