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PARABOLIC STARLIKE AND UNIFORMLY CONVEX FUNCTIONS ASSOCIATED WITH POISSON DISTRIBUTION SERIES

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Abstract

The purpose of the present paper is to establish connections between various subclasses of analytic univalent functions by applying a certain convolution operator involving Poisson distribution series. To be more precise,we investigate such connections with the classes of parabolic starlike and uniformly convex functions associated with Poisson distribution series .

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1 Introduction

Let $\mathcal{A}$ be the class of analytic functions in the unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad z \in \mathbb{U}. \tag{1}$$

We also let $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of functions which are normalized by $f(0) = 0 = f'(0) - 1$ and also univalent in $\mathbb{U}$. Denote by $\mathcal{T}$ the subclass of $\mathcal{A}$ consisting of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0. \tag{2}$$

For functions $f \in \mathcal{A}$ given by (1) and $g \in \mathcal{A}$ given by $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$, we define the Hadamard product (or convolution) of f and g by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad z \in \mathbb{U}.$$

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A function $f \in \mathcal{A}$ is said to be starlike of order α $(0 \leq \alpha < 1)$, if and only if

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha \quad (z \in \mathbb{U}).$$

This function class is denoted by $\mathcal{S}^*(\alpha)$. We also write $\mathcal{S}^*(0) =: \mathcal{S}^*$, where $\mathcal{S}^*$ denotes the class of functions $f \in \mathcal{A}$ such that $f(\mathbb{U})$ is starlike with respect to the origin.

A function $f \in \mathcal{A}$ is said to be convex of order α $(0 \leq \alpha < 1)$ if and only if

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha \quad (z \in \mathbb{U}).$$

This class is denoted by $\mathcal{K}(\alpha)$. Further, $\mathcal{K} = \mathcal{K}(0)$, the well-known standard class of convex functions.

It is an established fact that $f \in \mathcal{K}(\alpha) \Longleftrightarrow zf' \in \mathcal{S}^*(\alpha)$.

We consider the following subclasses $\mathcal{P}_\lambda(\gamma, \beta)$ and $\mathcal{Q}_\lambda(\gamma, \beta)$ of analytic functions studied by Ali et al., [1] and Murugusundaramoorthy et al., [12].

For some γ $(0 \leq \gamma < 1)$, λ $(0 \leq \lambda \leq 1)$, $\beta \geq 0$ and functions of the form (1), we let $\mathcal{P}_\lambda(\gamma, \beta)$ be the subclass of $\mathcal{S}$ satisfying the analytic criteria

$$\Re\left(\frac{zf'(z)}{(1-\lambda)z + \lambda f(z)} - \gamma\right) > \beta\left|\frac{zf'(z)}{(1-\lambda)z + \lambda f(z)} - 1\right|, \ z \in \mathbb{U}$$

and also let $\mathcal{Q}_\lambda(\gamma, \beta)$, be the subclass of $\mathcal{S}$ satisfying the analytic criteria

$$\Re\left(\frac{zf'(z) + z^2 f''(z)}{(1-\lambda)z + \lambda zf'(z)} - \gamma\right) > \beta\left|\frac{zf'(z) + z^2 f''(z)}{(1-\lambda)z + \lambda zf'(z)} - 1\right|, \ z \in \mathbb{U}.$$

Also denote $\mathcal{P}^*_\lambda(\gamma, \beta) = \mathcal{P}_\lambda(\gamma, \beta) \cap \mathcal{T}$ and $\mathcal{Q}^*_\lambda(\gamma, \beta) = \mathcal{Q}_\lambda(\gamma, \beta) \cap \mathcal{T}$, the subclasses of $\mathcal{T}$.

Example 1. *[5, 18] For some γ $(0 \leq \gamma < 1)$, $\beta \geq 0$ and choosing $\lambda = 1$ and functions of the form (2), we let $\mathcal{P}^*_1(\gamma, \beta) \equiv \mathcal{TS}_\mathcal{P}(\gamma, \beta)$ be the subclass of $\mathcal{S}$ satisfying the analytic criteria*

$$\Re\left(\frac{zf'(z)}{f(z)} - \gamma\right) > \beta\left|\frac{zf'(z)}{f(z)} - 1\right| \ z \in \mathbb{U}.$$

and also let $\mathcal{Q}^*_1(\gamma, \beta) \equiv \mathcal{UCT}(\gamma, \beta)$ the subclass of $\mathcal{S}$ satisfying the analytic criteria

$$\Re\left(1 + \frac{zf''(z)}{f'(z)} - \gamma\right) > \beta\left|\frac{zf''(z)}{f'(z)}\right|, \ z \in \mathbb{U}.$$

By taking $\gamma = 0$ in the above example we get the subclasses $\mathcal{TS}_\mathcal{P}(\beta)$ and $\mathcal{UCT}(\beta)$ studied in [17].

Example 2. *For some $\gamma(0 \le \gamma < 1)$, $\beta \ge 0$ and choosing $\lambda = 0$ and functions of the form (2),we let*
(i) $\mathcal{P}_0^*(\gamma,\beta) \equiv \mathcal{USD}(\gamma,\beta)$ *the subclass of* $\mathcal{S}$ *satisfying the analytic criteria*

$$\Re\left(f'(z)-\gamma\right) > \beta\left|f'(z)-1\right| \;\; z \in \mathbb{U}.$$

and
(ii) $\mathcal{Q}_0^*(\gamma,\beta) \equiv \mathcal{UCD}(\gamma,\beta)$ *the subclass of* $\mathcal{S}$ *satisfying the analytic criteria*

$$\Re\left((zf'(z))'-\gamma\right) > \beta\left|(zf'(z))'-1\right|, \;\; z \in \mathbb{U}.$$

Murugusundaramoorthy et al., [12] have studied $\mathcal{P}_\lambda(\gamma,\beta)$ and $\mathcal{Q}_\lambda(\gamma,\beta)$ based on Hurwitz-zeta functions and have obtained coefficient bound for functions in this generalized class and further discussed some inclusion properties. We state the following necessary and sufficient conditions for functions $f \in \mathcal{P}_\lambda^*(\gamma,\beta)$, $f \in \mathcal{Q}_\lambda^*(\gamma,\beta)$ and the subclasses stated in the above examples.(a special case given in [12].)

Lemma 1. *A function $f \in \mathcal{T}$ and of the form* (2)
(i) belongs to the class $\mathcal{P}_\lambda^*(\gamma,\beta)$ if and only if

$$\sum_{n=2}^{\infty}[n(1+\beta)-\lambda(\gamma+\beta)]|a_n| \le 1-\gamma. \tag{3}$$

(ii) belongs to the class $\mathcal{Q}_\lambda^*(\gamma,\beta)$ if and only if

$$\sum_{n=2}^{\infty}n[n(1+\beta)-\lambda(\gamma+\beta)]|a_n| \le 1-\gamma. \tag{4}$$

Lemma 2. *[5, 18] A function $f \in \mathcal{T}$ and of the form* (2)
(i) belongs to the class $\mathcal{TS}_{\mathcal{P}}(\gamma,\beta)$ if and only if

$$\sum_{n=2}^{\infty}[n(1+\beta)-(\gamma+\beta)]|a_n| \le 1-\gamma.$$

(ii) belongs to the class $\mathcal{UCT}(\gamma,\beta)$ if and only if

$$\sum_{n=2}^{\infty}n[n(1+\beta)-(\gamma+\beta)]|a_n| \le 1-\gamma.$$

Lemma 3. *[12] A function $f \in \mathcal{T}$ and of the form* (2)
(i) belongs to the class $\mathcal{USD}(\gamma,\beta)$ if and only if

$$\sum_{n=2}^{\infty}n(1+\beta)|a_n| \le 1-\gamma.$$

(ii) belongs to the class $\mathcal{UCD}(\gamma,\beta)$ if and only if

$$\sum_{n=2}^{\infty}n^2(1+\beta)|a_n| \le 1-\gamma.$$

It is well known that the special functions (series) play an important role in geometric function theory, especially in the solution by de Branges of the famous Bieberbach conjecture. The surprising use of special functions (hypergeometric functions) has prompted renewed interest in function theory in the last few decades. There is an extensive literature dealing with geometric properties of different types of special functions, especially for the generalized, Gaussian hypergeometric functions [6, 7, 15, 19, 20] and the Bessel functions [2, 3, 8].

A variable x is said to be Poisson distribution if it takes the values $0, 1, 2, 3, \ldots$ with probabilities $e^{-m}, m\frac{e^{-m}}{1!}, m^2\frac{e^{-m}}{2!}, m^3\frac{e^{-m}}{3!}, \ldots$ respectively, where m is called the parameter. Thus

$$P(x = k) = \frac{m^k\, e^{-m}}{k!}, k = 0, 1, 2, 3, \ldots$$

Very recently, Porwal[14] (also see[10, 13]) has introduce a power series whose coefficients are probabilities of Poisson distribution

$$\Phi(m, z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n, \qquad z \in \mathbb{U} \tag{5}$$

and we note that, by ratio test the radius of convergence of the above series is infinity. In [14], Porwal also defined the series

$$\Psi(m, z) = 2z - \Phi(m, z) = z - \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n, \qquad z \in \mathbb{U}. \tag{6}$$

Now, we considered the linear operator

$$\mathcal{I}(m, z) : \mathcal{A} \to \mathcal{A}$$

defined by the convolution or Hadamard product

$$\mathcal{I}(m, z)f = \Phi(m, z) * f(z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} a_n z^n. \tag{7}$$

The purpose of the present paper is to establish connections between distribution function and Geometric Function Theory. Motivated by result on connections between various subclasses of analytic univalent functions by using hypergeometric functions [6, 7, 15, 19, 20] and generalized Bessel functions [2, 3, 11, 8] we establish a number of connections between the classes $\mathcal{P}^*_\lambda(\gamma, \beta)$ and $\mathcal{Q}^*_\lambda(\gamma, \beta)$ by applying the convolution operator given by (7).

2 Main results

For convenience throughout in the sequel, we use the following notations:

$$\sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} = e^m - 1 \tag{8}$$

$$\sum_{n=j}^{\infty} \frac{m^{n-1}}{(n-j)!} = m^{j-1} e^m, \; j \geq 2 \tag{9}$$

Theorem 1. *If* $m > 0$ *then* $\Psi(m, z)$, *is in the class* $\mathcal{P}^*_\lambda(\gamma, \beta)$ *if and only if*

$$(1+\beta)m + [(1+\beta) - \lambda(\gamma+\beta)](1 - e^{-m}) \leq 1 - \gamma. \tag{10}$$

Proof. Since

$$\Psi(m, z) = z - \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n,$$

by virtue of Lemma 1 and (3) it suffices to show that

$$\sum_{n=2}^{\infty} [n(1+\beta) - \lambda(\gamma+\beta)] \frac{m^{n-1}}{(n-1)!} e^{-m} \leq 1 - \gamma.$$

Now by writing $n = (n-1) + 1$ we get

$$\begin{aligned}
& (1+\beta) \sum_{n=2}^{\infty} n \frac{m^{n-1}}{(n-1)!} e^{-m} - \lambda(\gamma+\beta) \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} \\
= \quad & (1+\beta) \sum_{n=2}^{\infty} (n-1) \frac{m^{n-1}}{(n-1)!} e^{-m} + [(1+\beta) - \lambda(\gamma+\beta)] \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} \\
= \quad & (1+\beta) \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-2)!} e^{-m} + [(1+\beta) - \lambda(\gamma+\beta)] \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} \\
\leq \quad & (1+\beta)m + [(1+\beta) - \lambda(\gamma+\beta)](1 - e^{-m}).
\end{aligned}$$

But this expression is bounded above by $1 - \gamma$ if and only if (10) holds. Thus the proof is complete. □

Theorem 2. *If* $m > 0$ *then* $\Psi(m, z)$, *is in the class* $\mathcal{Q}^*_\lambda(\gamma, \beta)$ *if and only if*

$$(1+\beta)m^2 + [3(1+\beta) - \lambda(\gamma+\beta)]m + [(1+\beta) - \lambda(\gamma+\beta)](1 - e^{-m}) \leq 1 - \gamma. \tag{11}$$

Proof. Since

$$\Psi(m, z) = z - \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} z^n,$$

by virtue of Lemma 1 and (4) it suffices to show that

$$\sum_{n=2}^{\infty} n[n(1+\beta) - \lambda(\gamma+\beta)] \frac{m^{n-1}}{(n-1)!} e^{-m} \leq 1 - \gamma.$$

Let

$$L(\gamma, \beta) = \sum_{n=2}^{\infty} \left(n^2(1+\beta) - n\lambda(\gamma+\beta)\right) \frac{m^{n-1}}{(n-1)!} e^{-m}.$$

Writing $n = (n-1)+1$ and $n^2 = (n-1)(n-2)+3(n-1)+1$, we can rewrite the above term as

$$\begin{aligned}
L(\gamma,\beta) &= (1+\beta)\sum_{n=2}^{\infty}(n-1)(n-2)\frac{m^{n-1}}{(n-1)!}e^{-m} \\
&+ [3(1+\beta)-\lambda(\gamma+\beta)]\sum_{n=2}^{\infty}(n-1)\frac{m^{n-1}}{(n-1)!}e^{-m} \\
&+ [(1+\beta)-\lambda(\gamma+\beta)]\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m} \\
&= (1+\beta)\sum_{n=3}^{\infty}\frac{m^{n-1}}{(n-3)!}e^{-m} \\
&+ [3(1+\beta)-\lambda(\gamma+\beta)]\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-2)!}e^{-m} \\
&+ [(1+\beta)-\lambda(\gamma+\beta)]\sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}e^{-m}
\end{aligned}$$

$$= (1+\beta)m^2 + [3(1+\beta)-\lambda(\gamma+\beta)]m + [(1+\beta)-\lambda(\gamma+\beta)](1-e^{-m}).$$

But this expression is bounded above by $1-\gamma$ if and only if (11) holds. Thus the proof is complete. □

Corollary 1. *If $m > 0$ then $\Psi(m,z)$, is in the class*
(i) is in the class $\mathcal{TS}_{\mathcal{P}}(\gamma,\beta)$ if and only if

$$(1+\beta)me^m \le 1-\gamma,$$

(ii) is in the class $\mathcal{UCT}(\gamma,\beta)$ if and only if

$$e^m[(1+\beta)m^2 + [3+2\beta-\gamma)]m] \le 1-\gamma.$$

Corollary 2. *If $m > 0$ then $\Psi(m,z)$, is in the class*
(i) is in the class $\mathcal{USD}(\gamma,\beta)$ if and only if

$$(1+\beta)(m+1-e^{-m}) \le 1-\gamma,$$

(ii) is in the class $\mathcal{UCD}(\gamma,\beta)$ if and only if

$$(1+\beta)m^2 + 3(1+\beta)m + (1+\beta)(1-e^{-m}) \le 1-\gamma.$$

3 Inclusion Properties

A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{R}^{\tau}(\mu,\delta)$, $(\tau \in \mathbb{C}\backslash\{0\},\ 0 < \mu \le 1; \delta < 1)$, if it satisfies the inequality

$$\left|\frac{(1-\mu)\frac{f(z)}{z} + \mu f'(z) - 1}{2\tau(1-\delta) + (1-\mu)\frac{f(z)}{z} + \mu f'(z) - 1}\right| < 1, \quad (z \in \mathbb{U}).$$

The class $\mathcal{R}^\tau(\mu,\delta)$ was introduced earlier by Swaminathan [20](for special cases see the references cited there in) and obtained the following estimate.

Lemma 4. *[20] If $f \in \mathcal{R}^\tau(\mu,\delta)$ is of form (1), then*

$$|a_n| \leq \frac{2\,|\tau|\,(1-\delta)}{1+\mu(n-1)}, \quad n \in \mathbb{N}\setminus\{1\}. \tag{12}$$

The bounds given in (12) are sharp.

Making use of the Lemma4, we will study the action of the Poisson distribution series on the class $\mathcal{Q}^*_\lambda(\gamma,\beta)$ in the following theorem.

Theorem 3. *If $m > 0$ and $f \in \mathcal{R}^\tau(\mu,\delta)$, if the inequality*

$$\Big[\,(1+\beta)m + [(1+\beta)-\lambda(\gamma+\beta)](1-e^{-m})\Big] \leq \frac{\mu(1-\gamma)}{2\,|\tau|\,(1-\delta)} \tag{13}$$

is satisfied, then $\mathcal{I}(m,z)f \in \mathcal{Q}^*_\lambda(\alpha,\beta)$.

Proof. Let f be of the form (1) belong to the class $\mathcal{R}^\tau(\mu,\delta)$. By virtue of Lemma 1 and (4) it suffices to show that

$$\sum_{n=2}^{\infty} n[n(1+\beta)-\lambda(\gamma+\beta)]\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n| \leq 1-\gamma.$$

Since $f \in \mathcal{R}^\tau(\mu,\delta)$ then by Lemma 12 we have

$$|a_n| \leq \frac{2\,|\tau|\,(1-\delta)}{1+\mu(n-1)}, \quad n \in \mathbb{N}\setminus\{1\}.$$

$$\text{Hence } L(m,\gamma,\beta) = \sum_{n=2}^{\infty} n[n(1+\beta)-\lambda(\gamma+\beta)]\frac{m^{n-1}}{(n-1)!}e^{-m}|a_n|$$

$$\leq \quad 2\,|\tau|\,(1-\delta)\sum_{n=2}^{\infty} n\frac{[n(1+\beta)-\lambda(\gamma+\beta)]}{1+\mu(n-1)}\frac{m^{n-1}}{(n-1)!}e^{-m}.$$

Since $1+\mu(n-1) \geq n\mu$, we get

$$L(m,\gamma,\beta) \quad \leq \quad \frac{2\,|\tau|\,(1-\delta)}{\mu}\sum_{n=2}^{\infty}[n(1+\beta)-\lambda(\gamma+\beta)]\frac{m^{n-1}}{(n-1)!}e^{-m}.$$

Proceeding as in Theorem 2, we get

$$L(m,\gamma,\beta) \leq \frac{2\,|\tau|\,(1-\delta)}{\mu}\Big[\,(1+\beta)m + [(1+\beta)-\lambda(\gamma+\beta)](1-e^{-m})\Big].$$

But this expression is bounded above by $1-\gamma$ if and only if (15) holds. Thus the proof is complete. □

Corollary 3. *If $m > 0$ and $f \in \mathcal{R}^{\tau}(\mu, \delta)$, if the inequality*

$$\Big[(1+\beta)(m+1-e^{-m}) \Big] \leq \frac{\mu(1-\gamma)}{2\,|\tau|\,(1-\delta)} \tag{14}$$

is satisfied, then $\mathcal{I}(m,z)f \in \mathcal{UCD}(\gamma, \beta)$.

Corollary 4. *If $m > 0$ and $f \in \mathcal{R}^{\tau}(\mu, \delta)$, if the inequality*

$$\left[\frac{(1+\beta)2\,|\tau|\,(1-\delta)m}{\mu - 2(1-e^{-m})\,|\tau|\,(1-\delta)} \right] \leq 1-\gamma \tag{15}$$

is satisfied, then $\mathcal{I}(m,z)f \in \mathcal{UCT}(\gamma, \beta)$.

Remark 1. *The above conditions are also necessary for functions $\Psi(m,z)$ of the form(6).*

Theorem 4. *Let $m > 0$, then $\mathcal{L}(m,z) = \int_0^z \frac{\mathcal{I}(m,t)}{t} dt$ is belongs to the class $\mathcal{Q}_{\lambda}^{*}(\gamma, \beta)$ if and only if*

$$(1+\beta)m + [(1+\beta) - \lambda(\gamma+\beta)](1-e^{-m}) \leq 1-\gamma. \tag{16}$$

Proof. Since

$$\mathcal{L}(m,z) = z - \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n-1)!} e^{-m} \frac{z^n}{n}$$

then by Theorem 2 we need only to show that

$$\sum_{n=2}^{\infty} n[n(1+\beta) - \lambda(\gamma+\beta)] \frac{1}{n} \frac{m^{n-1}}{(n-1)!} e^{-m} \leq 1-\gamma.$$

That is,

$$\sum_{n=2}^{\infty} [n(1+\beta) - \lambda(\gamma+\beta)] \, \frac{m^{n-1}}{(n-1)!} e^{-m} \quad \leq \quad 1-\gamma.$$

Now by writing $n = (n-1)+1$ and Proceeding as in Theorem 2, we get

$$\sum_{n=2}^{\infty} n[n(1+\beta) - \lambda(\gamma+\beta)] \, \frac{m^{n-1}}{(n-1)!} e^{-m} = (1+\beta)m + [(1+\beta) - \lambda(\gamma+\beta)](1-e^{-m})$$

which is bounded above by $1-\gamma$ if and only if (16) holds. □

Corollary 5. *Let $m > 0$, then $\mathcal{L}(m,z) = \int_0^z \frac{\mathcal{I}(m,t)}{t} dt$ is belongs to the class $\mathcal{UCT}(\gamma, \beta)$ if and only if*

$$(1+\beta)me^{m} \leq 1-\gamma.$$

Corollary 6. *Let $m > 0$, then $\mathcal{L}(m,z) = \int_0^z \frac{\mathcal{I}(m,t)}{t}dt$ is belongs to the class $\mathcal{UCD}(\gamma,\beta)$ if and only if*

$$(1+\beta)[m+1-e^m] \leq 1-\gamma.$$

Concluding Remark: By taking $\gamma = 0$ and specializing $\lambda = 0$ or $\lambda = 1$ one can deduce the above results for various subclasses analogous to the classes studied in [17].Further,by taking $\beta = 0$ and specializing $\lambda = 0$ or $\lambda = 1$ one can deduce the above results for various subclasses analogous to the classes studied in [16].

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