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Decisions in uncertainty based on entropy

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Abstract: *At present, the choice of the best solutions out of many possible under conditions of uncertainty is the actual economic task, arising and to be solved in many economic situations. Famous classical approaches to its solution are based on various assessments of decision-making practical situations. However, they often give insufficiently accurate or incorrect results, and do not satisfy sustainability requirements, when the only invariant calculation result relative to calculation methodology is a reliable one and a corresponding to the reality result. This article describes an alternative approach to the justification of decisions under conditions of uncertainty without the construction and use of assumptions about the decision-making situation and in conformity with the approaches of the stability theory. The problem of multi-criteria decision-making in conditions of complete uncertainty, wherein structuring of alternatives is performed using the fuzzy entropy, has been formulated and conceptually investigated. The idea of the described method assumes that the criterial conformity is estimated by fuzzy numbers and (or) linguistic allegations, i.e. formalizes by tools of fuzzy set theory. In opposition to classical approaches, this approach does not require the construction of hypotheses about the possible circumstances of decision-making and meets the requirements of stability theory. As a confirmation, it has been shown that the calculation of the fuzzy entropy by various methods does not lead to contradictory results. In this work appropriateness and practicality of using fuzzy entropy criterion for ordering sets of alternatives in fuzzy conditions of decision making has been is substantiated. The method for calculating the fuzzy entropy when evaluating criteria in linguistic form has been grounded. The paper presents numerical examples for which the fuzzy entropy calculation allows generating grounded clear recommendations and choose the best solution, which does not provide, under the given concrete numeric data, classical methods. The proposed approach for ordering and search of alternative solutions with a strong uncertainty using fuzzy entropy makes it possible to significantly enhance the validity of the required multi-criteria decision through the achievement of the invariance of the calculation results regarding the models and methods of processing fuzzy input data.*

Key-words: *multi-criteria decisions, fuzzy sets applying, fuzzy entropy, alternatives fuzzy ordering, fuzzy modelling*

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1. Introduction

The situations of the necessity of multicriteria decisions are quite frequent in various areas of economics: management and business, financial, investment and banking, economic, industrial and trade activities (Balke and Nigel, 2014; Hudec et al, 2014; Janssen et al, 2017; Kostenko et al, 2014; Mastorakis and Siskos, 2016).

At the same time, in the context of globalization and competitive market relations, there is (and requires an objective assessment and reasonable choice) a sufficiently large number of diverse opportunities, options, alternatives (in any of the above-mentioned spheres of economic activity).Meanwhile, as a rule, the baseline information (according to which solutions must be built) is often insufficiently reliable, incomplete, poorly formalized and difficult to apply to traditional economic methods of classic statistical analysis (Diday et al, 1994; Malugin et al, 2014; Saaty and Ergu, 2015; Tomer, 2015; Zamani-Sabzi et al, 2016; Žmuk, 2017). Considering the aforementioned facts, it is highly desirable and necessary to implement and appropriately adapt various methods of the classical decision-making theory and tools to the economic tasks and problems nowadays.

In particular, such its use in the field of business are devoted publications (Canco, 2016; Kanagal, 2016; Kościelniak et al, 2015; Podviezko, 2015).

The applications in production activity was considered in research (Barbacioru, 2014; Rojas-Zerpa and Yusta, 2015).

The adaptation to management tasks was carried out in works (Puseljic et al., 2015; Sacheti et al., 2016).

The problems of multi-criteria decision-making for business in terms of the sets of alternatives was given in the articles (Gawlik, 2016; Iuan-Yuan Lu et al, 2016; Kitsios et al, 2015; Rezaei, 2015).

The corresponding approaches to solving logistical problems was described in studies (Aguezzoul, 2014; Dieaconescu et al, 2016; Erodlu, 2016; Olariu, 2014).

The possibilities for applying mentioned approaches in financial or banking sector was shown in papers (Forbes et al, 2015; Johnston et al, 2016; Karimi, 2014; Spãtãrelu and Petec, 2016).

In all these cases and tasks, the situations of full uncertainty occupy a special place, and the best mathematical tools for formalization, comprehensive review and consideration, and final effective solutions are represented by approaches based on the theory of fuzzy sets and fuzzy logic.

In the theory and practice of decision-making into a separate group, criteria for decision-making under conditions of total uncertainty stand out, when the decision maker faces a complete lack of information about the probabilities of states of the environment (nature), or this information cannot be considered as credible.

The uncertainty of such kind is called „hopeless” or „stupid” (Carrigan, 2010; Schjaer-Jacobsen, 2004; Sinn, 2012). The known methods of the structuring of alternatives commonly do not provide unequivocal solutions under conditions of full of uncertainty.

2. The basic methods for structuring alternatives

It is known that for making decisions in such conditions it is generally recommended to use the Wald, Laplace, Savage, Hurwitz criteria (Carrigan, 2010; Schjaer, 2004; Spătărelu and Petec, 2016).

It should be noted that in a situation of complete uncertainty, the theory does not provide unambiguous and mathematically rigorous recommendations on the selection criteria for the decision. In some sources, only general very vague considerations can be found on certain criteria, as shown in Table 1.

Wald criterion	Laplace criterion	Hurwitz criterion	Hodge-Lehmann criterion
The risk is not allowed. The calculations are carried out on the basis of the worst state of nature.	An equal probability of the states of nature is assumed, as there is no reliable information.	Nothing is known about the probabilities of states of nature A small number of decisions are implemented, some risk is allowed.	The probabilities of the states of nature are unknown, but there may be some assumptions about them. The solution theoretically allows an infinite number of realizations. Risk in a small number of implementations is allowed.

Source: universally recognized research classification

Table 1. *Conditions of use of the decision-making criteria*

In most sources, the description of the criteria in general is not accompanied by such information. Furthermore, in the practice of application of these criteria the cases when they are not able to uniquely regularize possible solutions are not rare. In this case, the application of several criteria to analyse the same situation cannot be accepted as correct because the conditions of using certain criteria are contradictory.

It appears that the major drawback of the above criteria is the contradiction between the declaration about the total uncertainty of decision-making conditions and the point estimates of the situation, on which some formal operations are carried out.

Besides, by introducing one or another hypothesis about the environment behaviour, we seem to remove the uncertainty, however, the hypothesis itself is just a guess, but not knowledge.

3. The fuzzy entropy as a criterion for structuring alternatives

Suppose some situation of decision-making is given by matrix:

$$M = \left\| m_{i,j} \right\|, \qquad (1)$$

where $i = \overline{i, I}$ - the number of possible alternative solutions A_i;

$j = \overline{j, J}$ - the number of states of the environment S_j;

$m_{i,j}$ - the result of applying solutions A_i when the condition of the environment S_j.

It should be noted that estimates m_{ij} have an expert character and therefore should be treated as fuzzy.

Let us choose the maximal element in the matrix (1):

$$m_{max}=max(m_{ij}) \text{ for all } i \text{ and } j. \qquad (2)$$

A measure of opportunity to achieve the maximum possible result, we define either

$$Pos_{ij}=min(m_{ij}/m_{max},1)=\mu_{ij}; \qquad (3)$$

or as:

$$Pos_{ij}=min\{(m_{max}-m_{ij})/m_{max},1)=\mu_{ij}. \qquad (4)$$

The use of the maximum element of the matrix (1) to compute the measure of possibilities on relations (3) or (4), but not the maximum value of the scale, in which matrix elements are evaluated, (1) can be explained as follows.

In the general case, depending on the particular task, matrix (1) may contain both components with both positive and negative values.

For further procedures, is comfortable to reduce matrix (1) to a positive form, but while allowing new values of the elements, we can go beyond the previously chosen scale. For example (Table 2), the original has the form where estimates belong to the scale [-10, 10]. (Tables 2 to 12 have as source own authors' numerical examples).

	S_1	S_2	S_3
A_1	-3	2	4
A_2	5	5	-7
A_3	3	8	-5

(Source: own authors' numerical example)

Table 2. *Example of initial matrix*

After bringing the elements to a positive form (Table 3), their values will belong to another scale, whose limits are necessary to be determined additionally.

	S_1	S_2	S_3
A_1	4	9	11
A_2	12	12	0
A_3	10	15	2

Table 3. *Reformed matrix*

Using the relation (2) irrespective of the type of the original matrix, the last procedure is not required. The value μ_{ij} can be considered as the value of the membership function of a fuzzy set, determining the linguistic meaning "the degree of deviation from the best possible result."

Because conversions (2) or (3) are performed on all the elements of the matrix (1), they do not make any changes to the situation to be analysed, only that instead of the matrix (1), we will analyse the matrix

$$M' = \left\| \mu_{i,j} \right\| \; . \tag{5}$$

It should be noted that the transition from the original matrix (1) to the matrix (5) does not distort the overall logic of the task because the kind of dominance relations for alternatives evaluation according to the criteria is retained.

Since the matrix (5) can be considered as a matrix of fuzzy values, characterizing a situation of uncertainty, then, for evaluating the alternative solutions, the fuzzy entropy can be used (Cavallaro et al., 2016; Jiuping et al., 2011; Yandong et al., 2016), which is determined either by the classical formula of Shannon:

$$H_i = -\sum_k \mu_{i,k} \log_2 \mu_{i,k} \; , \tag{6}$$

or by:

$$H_i = \left(\sum_j \mu_{i,j} \cap \overline{\mu_{i,j}}\right) / \left(\sum_j \mu_{i,j} \cup \overline{\mu_{i,j}}\right) , \tag{7}$$

where $\overline{\mu}_{i,j}$ - supplementation for $\mu_{i,j}$.

Comparing the ratios (3) and (4), it is easy to see that, when $m_{ij}=m_{max}$ from (3) it follows $Pos_{ij}=1$ and from (4) $Pos_{ij}=0$.

It should be noted that, despite the various values Pos_{ij} , the choice of expressions (3) or (4) does not influence the final result because, when using the entropy approach, the values $Pos_{ij}=1$ and $Pos_{ij}=0$ are equivalent and they both characterize conditions of full certainty.

An important circumstance for using equations (6) and (7) is that they do not assume the fulfillment of any special conditions of use, as it is the case for Wald and other criteria. Besides, they have entirely different and independent calculation algorithms which allow their use simultaneously for getting a more reasoned decision. Since entropy is an assessment of the level of uncertainty, then the best solution shall have a minimum value of entropy.

4. Numerical examples

The verification of the proposed approach to the choice of solutions under the conditions of total uncertainty was conducted for several different matrices of the form (1), randomly chosen from several sources. Given below are the results of testing on several „uncomfortable" cases (Table 4), where none of the known criteria has given a convincing result; at the same time, the entropy approach allowed us to find a solution.

M =

	S_1	S_2	S_3	S_4
A_1	7	5	3	10
A_2	5	3	8	4
A_3	5	3	4	2
A_4	8	5	3	10

Table 4. *An example of the payoff matrix*

Calculations based on Wald criterion pointed out as the best alternative A_1, A_2, A_4, Laplace criterion - A_4, Hurwitz (for $\alpha = 0.4$) - A_1 and A_4, Savage - A_3. The matrix (4) for the example has the next form (Table 5):

M' =

	S_1	S_2	S_3	S_4
A_1	0.7	0.5	0.3	1
A_2	0.5	0.3	0.8	0.4
A_3	0.5	0.3	0.4	0.2
A_4	0.8	0.5	0.3	1

Table 5. *Payment matrix is given by the ratio of* (3)

Fuzzy entropy values calculated by the formula (5) for an alternative A_1 = 1.38, A_2 = 1.81, A_3 = 2.36, A_4 = 1.28, on formula (6), accordingly, A_1 = 0.38, A_2 = 0.54, A_3 = 0.54, A_4 = 0.33, and the best alternative may be recognized A_4.

It is necessary, of course, to note that on alternative A_4 as the best alternative among others there are indications in the criteria of Wald, Laplace and Hurwitz. However, these solutions are not unambiguous, whereas the entropy criterion uniquely chose alternative A_4.

Let us consider another example of the payoff matrix (Table 6):

	S_1	S_2	S_3	S_4	S_5
A_1	5	6	4	6	9
A_2	6	5	3	4	8
A_3	7	6	6	7	5
A_4	6	7	5	4	3

Table 6. *Another example of the payoff matrix*

The results of the structuring according to criteria: Wald - $A_3 \succ A_1 \succ A_2 = A_4$, Laplace - $A_3 \succ A_1 \succ A_2 \succ A_4$, Savage - $A_1 \succ A_2 \succ A_3 \succ A_4$, Hurwitz (α=0.4) - $A_1 \succ A_3 \succ A_2 \succ A_4$, Shannon entropy - $A_3 \succ A_4 \succ A_2 \succ A_1$, Cosco entropy - $A_3 \succ A_4 \succ A_2 \succ A_1$.

One more example of the payoff matrix (Table 7):

	S_1	S_2	S_3	S_4	S_5
A_1	8	6	5	3	7
A_2	7	3	6	4	5
A_3	6	7	6	2	5
A_4	5	8	3	5	7

Table 7. *One more example of the payoff matrix*

The results of the structuring according to criteria: Wald - $A_1 = A_2 = A_4 \succ A_3$, Laplace - $A_1 \succ A_4 \succ A_3 \succ A_2$, Savage - $A_1 \succ A_3 = A_4 \succ A_2$, Hurwitz (α=0.4) - $A_1 = A_4 \succ A_2 \succ A_3$, Shannon entropy - $A_1 \succ A_4 \succ A_3 \succ A_2$, Cosco entropy - $A_1 \succ A_4 \succ A_3 \succ A_2$.

These examples also indicate that entropic criteria, unlike known ones, give unambiguous and stable solutions.

Another opportunity should also be noted for structuring alternatives on the basis of the entropy approach. Ii is known that, for fuzzy sets, the principle of excluding the third is not performed i.e. $\mu_{ij} \cap \overline{\mu}_{ij} \neq \varnothing$.

A nonzero value of this intersection can be regarded as an estimate of unresolved uncertainty; then, the alternative with the lowest value of this uncertainty can be regarded as the best. It is obvious that, in this situation, the integrated level of uncertainty may be determined by the entropy calculated by the ratio (6). The results of calculations have confirmed the resulting ranking of alternatives.

It should also be noted that, when calculating the ratio of (7) min-intersection and max-unification operations can be used, as well as the algebraic intersection and union (Luoh and Wang, 2001; Zhang, 2014).

In this case, the nature of the ranking of alternatives does not change. Let us note that the proposed approach corresponds to the methodology of the theory of stability, according to which the processing result must be invariant with respect to the method of processing.

5. Transformations to linguistic values and membership functions

When proceeding from operations on the elements of the payment matrix (1) to the transformation of their uncertainty estimates, the following approach can be used. In the definition domain of the payoff matrix values, it is possible to construct a term set of linguistic values $L=\{l_k:k=1,\ldots,K\}$ and the corresponding fuzzy sets $\mu_{lk}(z)$, $z \in [m_{\min} m_{\max}]$, where m_{min} and m_{max} presumed minimum and maximum values of the payoff matrix (1) (Figure1).

The number of linguistic values, type of membership functions and evaluations of alternatives will be determined by the character of the problem and the views of experts, involved in its solution.

When constructing membership functions, the direct method can be used (Luoh and Wang, 2001; Zhang, 2014), following the conditions set forth in (Aydin and Apaydin, 2008; Tošenovský et al., 2011), or the indirect one, for example the method of paired comparisons (Luoh and Wang, 2001; Zhang, 2014).

Figure 1 shows a possible variant of a set of linguistic values and the corresponding membership functions.

The triangular type of membership functions is chosen only for reasons of simplicity of the graphical representation. The set of linguistic values can obviously be different. In this case, it has only an illustrative character. It should be noted that the kind of membership functions will affect only the numerical values of the resulting estimates; at the same time, it does not affect the order of the alternatives ranking.

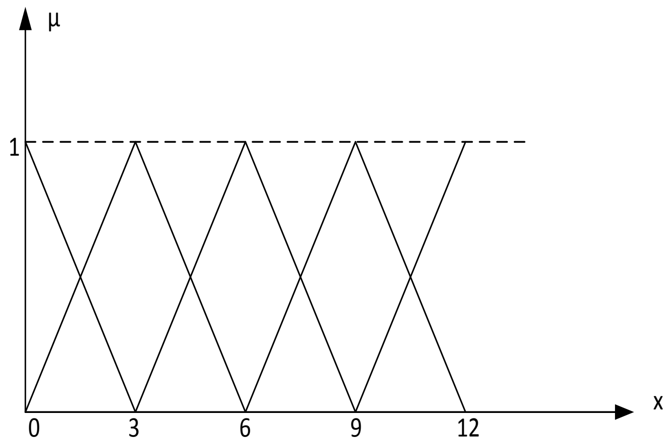

Fig. 1. *The membership functions*

The procedure of the transition from the numerical values of the matrix elements (1) to the linguistic ones is shown in Figure 2.

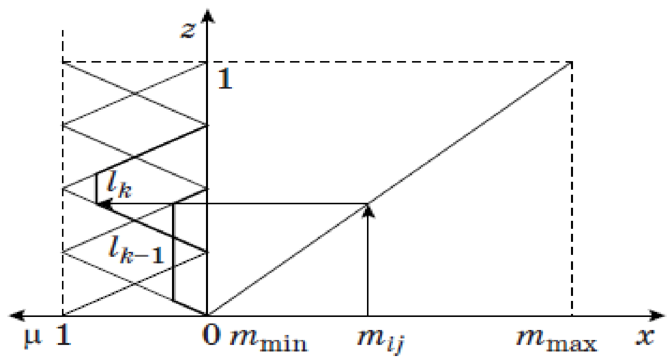

Fig. 2. *The transition from numerical to linguistic evaluations*

As seen from Figure 2, for the particular value m_{ij}, we have the inequality: $\mu_{l_k}(m_{ij}) > \mu_{l_{r-1}}(m_{ij})$.

If one interprets the values of the membership functions as the truth value of the corresponding linguistic meaning, $\mu_{l_k}(m_{ij})$ - this is an optimistic assessment of truthfulness, $\mu_{l_{r-1}}(m_{ij})$ - pessimistic, moreover, in addition, a combined estimate $l_k \vee l_{k-1}$ can also be constructed with membership function:

$$\mu_{l_k \cup l_{k-1}}(m_{ij}) = \mu_{l_k}(m_{ij}) \cup \mu_{l_{k-1}}(m_{ij}) = \max(\mu_{l_k}(m_{ij}), \mu_{l_{k-1}}(m_{ij})) . \tag{8}$$

Any of the above values can be used as elements of a matrix (5).

However, during the transition to linguistic values, it is possible to have a situation when for several completely different values m_{ij} will correspond different linguistic values, but with the same values of the corresponding membership functions. This may make decision-making impossible. Such a situation is presented in Table 8, whose elements are values of the respective membership functions during the transition from numerical estimates (Table 4) to linguistic ones, according to Figures 1 and 2.

	S1	S2	S3	S4
A1	0.66	0.66	1	0.66
A2	0.66	1	0.66	0.66
A3	0.66	1	0.66	0.66
A4	0.66	0.66	1	0.66

Table 8. *The values of membership functions*

If using optimistic or pessimistic assessments, it is possible to proceed as follows. By calculating the integral estimates $r_{l_k}(i,j)=\mu_{l_k}(m_{ij})\times m_{ij}$ or

$$r_{l_k-1}(i,j)=\mu_{l_k-1}(m_{ij})\times m_{ij} \text{ for all } m_{ij}, \tag{9}$$

and their normalized values

$$r^{н}_{l_k}(i)=r_{l_k}(i,j)/\max_{i,j} r_{l_k}(i,j) \tag{10}$$

or

$$r^{н}_{l_{k-1}}(i)=r_{l_{k-1}}(i,j)/\max_{i,j} r_{l_{k-1}}(i,j), \text{ for all } i \text{ and } j, \tag{11}$$

we will build either matrix $R^{н}_{l_k}=\left\|r^{н}_{l_k}(i,j)\right\|$ or $R^{н}_{l_k-1}=\left\|r^{н}_{l_k-1}(i,j)\right\|$ (Tables 9 and 10 respectively).

	S1	S2	S3	S4
A1	0,7	0,625	0,45	1
A2	0,625	0,45	0,8	0,4
A3	0,6875	0,45	0,4	0,2
A4	0,8	0,25	0,45	1

Table 9. *The normalized values of integral assessments obtained by the ratio* (10)

	S1	S2	S3	S4
A1	0,7	0,5	0,45	1
A2	0,5	0,45	0,8	0,4
A3	0,5	0,45	0,4	0,2
A4	0,8	0,5	0,45	1

Table 10. *The normalized values of integral assessments obtained by the ratio* (11)

In the considered case during the transition to linguistic scores, they obtain a representation in the form of fuzzy sets with trapezoidal membership functions. In this case, when calculating estimates $r_k(i,j)$ and $r_{k-1}(i,j)$ using equation (9), instead of values m_{ij} it is possible to use the evaluation of Chew- Park (Anshin et al, 2008):

$$C_p(i,j)=(a_1(i,j)+a_2(i,j)+a_3(i,j)+a_4(i,j))/4+w(a_2(i,j)+a_3(i,j))/2,$$

where $a_1(i,j)$, $a_4(i,j)$, $a_2(i,j)$,$a_3(i,j)$ - the coordinates of the upper and lower bases of the trapezoidal membership function.

According to (Luoh and Wang, 2001; Zhang, 2014), for the symmetric trapezoidal membership function, which is valid in our case, parameter *w* may be taken equal to one. If the combination of linguistic values is used, then, for the integrated assessment, it is necessary to find the generalized characteristic of the combination of linguistic assessments represented by the corresponding fuzzy set (8).

It is known that the generalized characteristic of a system of material points is the coordinate of the center of gravity. The membership functions of fuzzy sets (8) can be considered as a system of material points whose masses are equal to the values of the membership functions.

6. Obtaining integral linguistic estimations

Then the integral estimate of the form (9) using a combination of linguistic values (8) can be calculated as:

$$r_{l_k \cup l_{k-1}}(i,j) = \mu_{l_k \cup l_{k-1}}(CG_{l_k \cup l_{k-1}}) * CG_{l_k \cup l_{k-1}},$$

where $CG_{l_k \cup l_{k-1}}(i,j) = (\sum_{n_{ij}} \mu_{l_k \cup l_{k-1}}(z_{n_{ij}}) z_{n_{ij}}) / \sum_{n_{ij}} \mu_{l_k \cup l_{k-1}}(z_{n_{ij}})$ - coordinate of center of gravity for linguistic evaluations combination; $z_{n_{ij}} \in [m_{\min}, m_{\max}]$; $\mu_{l_k \cup l_{k-1}}(z_{r_{ij}})$; indexes *i* and *j* indicate that calculations are carried out for value m_{ij} (as shown in Figure 3).

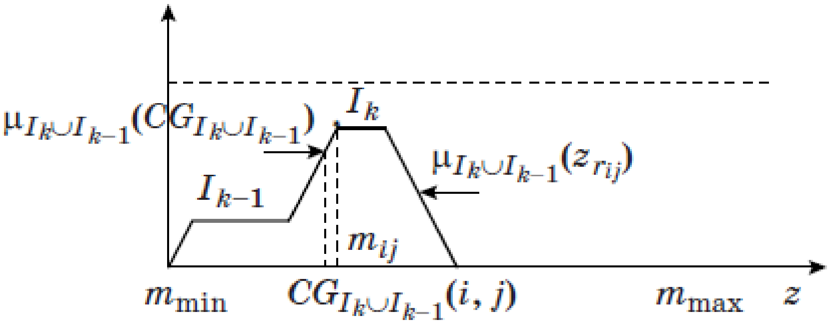

Fig. 3. *The calculation of the integral evaluation for the combination of linguistic values*

By normalizing the values $r_{l_k \cup l_{k-1}}(i, j)$ of the ratio (9), we form (see Table.11) $R^{н}_{l_k \cup l_{k-1}} = \left\| r^{н}_{l_k \cup l_{k-1}}(i, j) \right\|$ matrix where $r^{н}_{l_k \cup l_{k-1}}((i, j) = r_{l_k \cup l_{k-1}}(i, j) / \max_{i,j} r_{l_k \cup l_{k-1}}$

The values of $r^{н}_{l_k \cup l_{k-1}}(i, j)$ can be considered as an integral assessment of the uncertainty of achieving alternatives by evaluations at various states of nature for the maximum expected value.

To choose the best alternative, which must comply with the minimum level of integrated uncertainty, it is possible to use either e of the relationships (6) or (7), or both, simultaneously.

	S1	S2	S3	S4
A1	0,735471	0,507475	0,490677	1
A2	0,507475	0,490677	0,818237	0,447061
A3	0,507475	0,490677	0,447061	0,308472
A4	0,818237	0,507475	0,490677	1

Table 11. *Normalized evaluations when combining linguistic values*

In Table 12, the numbers in parentheses indicate a ratio number, on which the calculations were performed. Columns 1, 3 present the results of calculations, appropriate to the pessimistic assessment of the situation, in 2, 4 - optimistic, 5, 6 - results correspond to a combination of linguistic assessments.

An important circumstance should be noted. Table 12 shows the results of testing the entropic criteria at the transition from numerical estimates (provided by Table 4) to linguistic ones. In this case, the resulting structuring of alternatives completely coincides with that obtained by using only numeric estimates from the Table 4.

	$H(l_{k-1})$	$H(l_k)$	$H(l_k)$	$H(lk-1)$	$H(l_k U l_{k-1})$	$H(l_k U l_{k-1})$
Alternatives	(6)	(6)	(7)	(7)	(6)	(7)
	1	2	3	4	5	6
A_1	1.37	1.3	0.45	0.39	1.33	1.33
A_2	1.8	1.72	0.63	0.55	1.95	1.95
A_3	2.01	1.88	0.63	0.56	2.04	2.04
A_4	1.27	1.27	0.315	0.29	1.24	1.24

Table 12. *Normalized evaluations when combining linguistic values*

Also, one more circumstance should be noted. The matrix (1) may contain both positive assessments of the results of the solutions choice and negative ones, which should be taken into account when writing the final ranking. So, for positive results, the ranking will be $A_4 \succ A_1 \succ A_2 \succ A_3$, for negative results - $A_3 \succ A_2 \succ A_1 \succ A_4$.

7. Conclusions

The proposed entropy approach to the choice of solutions under conditions of full uncertainty does not require additional terms, typical for known criteria.

The use of the entropy approach ensures full compliance with the theory of stability methodology, according to which only the result of data processing which is invariant under the processing method corresponds to reality.

At the same time, using the known criteria, the result of processing depends on the processing method and reflects the subjectivity of the researcher rather than objective relations.

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