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ON SOME METHODS OF ACHIEVING A CONTINUOUS AND DIFFERENTIATED ASSESSMENT IN *LINEAR ALGEBRA AND ANALYTIC AND DIFFERENTIAL GEOMETRY* COURSES AND SEMINARS

Monica A.P. PURCARU[1]

Abstract: *This paper aims at highlighting some aspects related to assessment as regards its use as a differentiated training strategy for Linear Algebra and Analytic and Differential Geometry courses and seminars. Thus, the following methods of continuous differentiated assessment are analyzed and exemplified: the portfolio, the role play, some interactive methods and practical examinations.*

Key words: *differentiated assessment, portfolio, role play, interactive methods, practical examinations*

1. Introduction

Eliminating the admission examination at most universities has created situations where courses reunite from graduates of art high schools to laureates of school Olympics. Presenting attractive courses under these conditions is a challenge beyond scientific research (Căuş, 2015). Following the previous idea, we believe that at least for the continuous assessment in the first few weeks (sometimes even during the whole year, depending on the occasion) the teacher must also find ways to differentiate the assessment.

"The assessment method is, in fact, the shared teacher-student pathway leading to the implementation of any evaluative approach in order to collect information about the process and the product of learning, processing and capitalizing on various purposes." (Manolescu, 2010, p.159).

Differentiation and individualization of education is an old pedagogical issue, but always current, because people differ from one another not only in their way of thinking and being, but also in their ability and rhythm of learning, and in their attitude towards it (Jinga, 2005).

Differentiated work with students in Mathematics courses and seminars means the choice of scientific contents, training strategies appropriate to their teaching – learning – assessment, according to the possibilities and particularities of the students.

[1] *Transilvania* University of Braşov, mpurcaru@unitbv.ro

"The individualized work consists not in the individual execution of the same work by all students, but in electing the specific work adapted to each student" (Dottreus, 1970).

Differentiation and individualization of teaching-learning strategies is closely related to the differentiation and individualization of assessment strategies.

The assessment behaviour of the teacher plays a key role in giving students feedback on their performance and enhancing learning motivation (Cocoradă, Luca, Pavalache-Ilie, 2009).

In the last decades, evaluation has become one of the major issues in education. Multiple changes in social, political, economic, etc. conceptions determine changes in the evaluation plan (Voiculescu, Bocoş, Potolea, Cucoş, Cozma, & Marinescu, 2013).

By diversifying and applying differentiated forms of ongoing assessment, school progress can be significantly influenced (Junc, 2015). This can be done in various ways, knowing that "there are no templates, no binding prescriptions, but there will be the need, in exchange, for talent and the desire to discover the vocation for the profession of Mathematics teacher (Vălcan, 2013).

2. Methods to achieve a continuous and differentiated assessment in *Linear Algebra and Analytic and Differential Geometry* courses and seminars

Due to the large differences in students' knowledge of Algebra and Geometry at the moment of their admission to university, the differentiated use of alternative methods for continuous assessment in Mathematics is absolutely necessary, in addition to traditional assessment methods (oral or written examinations at the end of semester), contributing thus to students' progress in the above mentioned disciplines. Finding ways both to differentiate the assessment of students during the semester and to score their contributions will motivate them in learning, leading implicitly to the improvement of their performances in the discussed disciplines. The successful outcome of a seminar is directly dependent upon the assessment made by the teacher during the course.

In the case of teaching mathematical notions at the use of differentiated scientific contents with or without differentiated teaching strategies, it is natural that the assessment should be done differentiated as well.

Dividing students into level groups can be done in several ways. One of these ways may be adapted as specified in the quotation below.

"In the case of taking over new classes of students, as it is the case of the 5th grade, it is recommended that the teacher should give 2-3 tests at the beginning of the school year in order to establish the knowledge level of the pupil in relation to the level of his / her own requirements. A differentiation of the tests based on different indicators, such as knowledge, computation or perspicacity, helps to outline a preliminary classification that will either be confirmed or invalidated by ulterior current activities" (Banea, 1998, p.179).

The differentiated use of assessment methods in *Linear Algebra and Analytic and Differential Geometry* courses and seminars, where differentiated training strategies have been chosen, can be done in several ways, all pointing out to the idea of obtaining a more thorough feedback, which should allow the teacher to intervene promptly in helping progress both the high-level and the low-level students.

We will further present some of these ways, noting that differentiated assessment in the mentioned seminars and courses can be designed in definitely much more ways than the ones discussed below, being dictated by the needs of each group of students:

a) Differentiated assessment can be achieved through a portfolio.

b) Differentiated assessment can be achieved through an interactive method.

c) Differentiated assessment can be achieved through practical tests.

The continuous and differentiated assessment styles forms listed above will be onwards analyzed and exemplified.

3. Achieving continuous and differentiated assessment of students' knowledge by means of portfolio

From among the alternative assessment methods, we will now exemplify a differentiated assessment method achieved by means of portfolio.

In *Linear Algebra and Analytic and Differential Geometry* courses and seminars, the requirements for the preparation of the portfolio are given in a differentiated manner.

Thus, for the first item of the portfolio, students were given to solve to a certain deadline, a set of problems with a difficulty degree corresponding to their level of training in Algebra and Geometry.

The second item of the portfolio was a project, having a differentiated subject as follows: for the students falling behind in these branches of Mathematics, the task was to prepare two abstracts about the conics on canonical equations and respectively, the straight line in plane, both subjects being studied in high school; for the middle-level students, the task was to study a theoretical subject, complementary to the theory taught in the course, such as "Plane curves often used in the technique: definition, properties (without proofs)"; and finally, for the high-level students, the theme of the project was to study the "Titeica and Bertran curves: definition, properties (with proofs)". The structure of the project, as well as the way of its presentation (oral, poster, based on questions) was left at the latitude of each student.

The third item of the portfolio was also given in a differentiated manner. The students were asked to come up with new, different proofs for two problems (in *Linear Algebra*, respectively *Analytic Geometry*, chosen in a differentiated manner according to each level of knowledge), which were already solved in one way at the seminar together with the teacher.

The grade for the portfolio represents 10 % of the final mark in the *Linear Algebra and Analytic and Differential Geometry* discipline.

4. Achieving continuous and differentiated assessment of students' knowledge by means of interactive methods

Using some variants of interactive methods as differentiated assessment strategies in the courses and seminars of *Linear Algebra and Analytic and Differential Geometry* discipline, will lead to removing most of the obstacles to progress, both for the students falling behind, as well as for the other students, since the professor can more accurately spot the students gaps in this discipline.

Besides the examples below, there are also other ways to differentiate in the assessment process by using variants of known interactive methods, which will be chosen according

to the needs identified in each group of students, and depending on the learning style of each student.

Some differentiated assessment examples using this method for the *Linear Algebra and Analytic and Differential Geometry* courses and seminars are presented below.

Firstly we illustrate a variant of differentiated use, with the purpose of the continuous assessment of students' knowledge in the *Analytic Geometry* seminar, having the following subject: "The Euclidian Space of Free Vectors", using a variant of the "Tour of the Gallery" method. The teacher prepares two worksheets – one with low difficulty problems, and the other with higher level problems – distributing them to the students according to their level of knowledge in *Linear Algebra and Analytic and Differential Geometry*, asking them to group by the letter of the received worksheet. Thus, two groups of students are formed, having different levels of knowledge in the above mentioned discipline, and receiving the worksheets below.

Sheet A (Lower Level)

1. The vectors are given:

$\bar{a}$, $\bar{b}$ and $\bar{c}$ with $\|\bar{a}\|=1$, $\|\bar{b}\|=2$, $\|\bar{c}\|=3$, $\sphericalangle(\bar{a},\bar{b})=\frac{\pi}{3}$, $\sphericalangle(\bar{a},\bar{c})=\frac{\pi}{4}$, $\sphericalangle(\bar{b},\bar{c})=\frac{\pi}{6}$. Calculate the vector norm $\bar{a}+\bar{b}-\bar{c}$.

2. Let: $\bar{r}_A=2\bar{i}+\bar{j}+\bar{k}$, $\bar{r}_B=-5\bar{k}$, $\bar{r}_C=\bar{i}+3\bar{j}-\bar{k}$, $\bar{r}_D=\bar{i}-\bar{j}+2\bar{k}$, be the position vectors of points A, B, C and D. Determine: the scalar product of the vectors $\overline{AB}$ and $\overline{AC}$, the vector product of the vectors $\overline{AB}$ and $\overline{AC}$ and the mixed product of the vectors $\overline{AB}$, $\overline{AC}$, $\overline{AD}$.

Sheet B (Higher Level)

1. The vectors are given:

$\bar{a}$, $\bar{b}$ şi $\bar{c}$ cu $\|\bar{a}\|=1$, $\|\bar{b}\|=2$, $\|\bar{c}\|=3$, $\sphericalangle(\bar{a},\bar{b})=\frac{\pi}{3}$, $\sphericalangle(\bar{a},\bar{c})=\frac{\pi}{4}$, $\sphericalangle(\bar{b},\bar{c})=\frac{\pi}{2}$. Calculate the parallelepiped volume built on the representatives by oriented segments to the common origin of the three free vectors.

2. Points A (1, -5,4), B (0, -3,1), C (-2, -4,3), D (4,4, -2) are the vertices of a tetrahedron. Calculate the height of the tetrahedron from vertex A.

Within each group, the students collaborate to solve the received worksheet and outline the found solution. After the 30-minute working time, one representative of each group displays the solution; the groups then analyze these solutions and correct them if necessary; in the end, each group reads the received comments.

Secondly, we exemplify a variant of the "R.A.I." method, with the purpose of differentiated oral assessment of first year students' knowledge in the *Linear Algebra* seminar, with the subject "Vector spaces, subspaces, operations with subspaces". Each student was asked to hand on a note to the teacher, containing a question personally chosen from the given learning unit.

Examples of questions asked by students, with varying degrees of difficulty are the following:

1. What algebraic structures learned in high school do you know? (Low level)
2. What is the meaning of the commutativity of a composition law? (Low level)
3. What is the connection between a field and a vector space? (Low level)
4. What is the meaning of the distributivity of scalar multiplication with respect to the addition of vectors? (Low level)
5. What operations with subspaces do you know? (Low level)
6. What is the null space? (Low level)
7. What is meant by the canonical basis of arithmetic space? (Low level)
8. What is a vector space? Exemplify. (Medium level)
9. What is meant by supplementary subspaces? (Medium level)
10. What is the subspace spanned by a nonempty subset of a vector space? (Medium level)
11. What is meant by a vector subspace? Exemplify. (Medium level)
12. What is meant by bases and dimension in a vector space? Exemplify. (Medium level)
13. Which is the theorem of the change of basis? (Medium level)
14. What supplementary subspaces do you know? (High level)
15. What theorem about the sum of two subspaces you know? Exemplify its use in a category of exercises. (High level)
16. What is the necessary and sufficient condition for the sum of the two subspaces to be direct? Exemplify a category of exercises whose sum is calculated with this theorem. (High level)
17. What theorem about the subspace dimension do you know and for what purpose can you use it? (High level)

When a student throws the ball to another student, the teacher will read a question from the previous list, corresponding to his level of training at *Linear Algebra*.

Thirdly, another example of differentiated assessment applied in the *Analytic Geometry* seminar with the help of a variant of the interactive method "Brainstorming" is given below.

The theme of the seminar was: "Plane and straight line in space". The group was divided into three homogeneous subgroups, depending on their level of knowledge in *Analytic Geometry*. In each subgroup, where the working tasks were appropriate for its level, this method was applied.

The teacher has chosen to solve three problems, of three different levels of difficulty, giving one to each subgroup and asking them to find as many ways to solve it during their 20-minute work.

Thus, while the lower level students received a problem in which only the straight line and plane equations, learned in the course, had to be applied, the problem of the middle-level students required the determination of the canonical equations of the intersection of

the two planes and the relative position of its opposite to another given straight line and plane. The problem of high-level students asked for the determination of the equations of some straight lines given in new, undisclosed conditions in the class. The subgroup of high-level students finally presented five different ways of solving the problem, the middle-level subgroup found two different ways to solve the problem received, and in the low-level subgroup several of the formulas learned were applied to write the required equations.

A fourth example of a differentiated assessment of the students' knowledge in *Linear Algebra*, accomplished with the help of a variant of the interactive method "Jigsaw" is described below. At the seminar on the subject "Eigenvalues, eigenvectors", after solving various problems in this unit of learning, in order to obtain the feedback, the group was divided into three homogeneous subgroups, depending on the knowledge of the students in *Linear Algebra*, each subgroup having the task of solving a problem from a subtopic of the expert worksheet, corresponding to his level of knowledge.

The expert worksheet for the seminar: "Eigenvalues, eigenvectors" is given below.

Subtopics of the expert worksheet:

1. Exercises to determine the eigenvalues of a matrix A or a linear transformation T. (Low level)
2. Exercises to determine the inverse matrix and the value of a matrix polynomial using the Hamilton-Cayley theorem. (Medium level)
3. Exercises to determine the diagonal form a matrix and the basis corresponding to the diagonal form. (High level)

The students of each subgroup have consulted each other after they have solved the task they have received, sending then a representative to the board to explain it and write it down.

5. Achieving continuous and differentiated assessment of students' knowledge by means of role play

The role play can be also used in the differentiated assessment of students' knowledge in Mathematics. Examples of continuous and differentiated assessment using this method, for the *Linear Algebra and Analytic and Differential Geometry* courses and seminars are presented below.

In order to increase the students' interest in the discipline of *Linear Algebra and Analytic and Differential Geometry*, the role play "Teacher for 20 minutes" was used during the continuous assessment. Thusly, the teacher – student was asked to present new problems in front of his fellow students, or even to present parts of theory.

For example, at the course with the subject: "Plane and straight line in space", a first year student at the Civil Engineering Faculty, who was also a Romanian language teacher at a high school, taught for 20 minutes the first subtopic of the course: "Different forms of the equation of a plane". She made use of her skills already formed in high school teaching and in this way she obtained a bonus to the final grade in this discipline.

Optional tasks also bring a bonus to the student's final grade in the *Linear Algebra and Analytic and Differential Geometry* discipline, which will motivate him / her to work with more interest throughout the semester.

6. Achieving continuous and differentiated assessment of students' knowledge by means of practical examinations

Practical examination is less used in the differentiated assessment of students' knowledge in Mathematics. It is recommended to be used by the teacher for a faster feedback regarding the level of understanding of the notions taught.

Differentiated assessment examples using this method for the *Linear Algebra and Analytic and Differential Geometry* courses and seminars are presented below.

A first example regards the *Analytic Geometry* seminar on the subject "Plane and straight line in space", where students were asked to demonstrate using a drawing on an A3 sheet and colored pencils, the formula: parallelepiped volume = 6 x the tetrahedron volume, as follows: lower-level students demonstrated this formula on the particular case of the rectangular parallelepiped, and students with a higher level of knowledge, demonstrated this formula on the general case of parallelepiped.

Another example from the *Analytic Geometry* course regards the topic "Quadrics". In order to increase the students' motivation to learn and to see the applicability of the taught subject, what was taught in the course was linked to situations encountered in real life. This was completed by presenting with video projector numerous buildings, bridges, railways, in which some of the quadrics were used. In order to achieve a continuous and differentiated assessment of the students' knowledge of quadrics, each student was asked to bring such examples in the next hour of the seminar to identify the quadrics learned. Each student identified two of the quadrics learned in the pictures and presented some of their properties as follows: for lower- level students the quadrics were: the sphere and the ellipsoid, for middle- level students the quadrics were: the cone and cylinders, and for higher level students, the quadrics were: the hyperboloid of one sheet and the hyperbolic paraboloid.

7. Conclusions

Continuous assessment of students' knowledge in the discipline *Linear Algebra and Analytic and Differential Geometry*, used as a differentiated training strategy, can be achieved by means of portfolio, role play and interactive methods or practical examinations, but most often with the help of worksheets given on group-level students.

The portfolio, the role play, and the interactive methods or practical examinations can serve as oral or written differentiated assessment methods in the discussed discipline, often used to seminars and rarely to courses. These are few ways of differentiated assessment, the choice of one or the other depending on both the existing situation in the classroom and on the talent of the teacher.

Continuous assessment cannot be permanently done differentiated, because, firstly, learning-teaching is not always done using the teaching-learning methods or didactic methods in a differentiated way, and secondly, the marking of the final low-level tests as well as high-level ones, presents some inconveniences, some limits, leading to the demobilization of students who have solid knowledge of Mathematics.

Other information may be obtained from the address: mpurcaru@unitbv.ro

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