# THE FIDELITY WITH WHICH THE DESIGN RULES REFLECTS THE ACTUAL BEHAVIOR AT SHEAR FORCE OF THE REINFORCED LIGHTWEIGHT CONCRETE ELEMENTS 

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#### Abstract

The paper presents a method for calculating lightweight reinforced concrete elements in inclined sections according to the current design norms. There are also described the principles and calculus relations to check the structural resistance of elements that require shear force reinforcement, respectively vertical reinforced elements. The experimental testing where conducted at the Reinforced and Prestressed Concrete Laboratory from Technical University of Cluj-Napoca, Faculty of Civil Engineering. Therefore, testing 16 beams of lightweight reinforced concrete have permitted the comparative study between the experimental values $\left(V_{R d}{ }^{e}{ }^{e}\right)$ and the calculated values $\left(V_{I R d}{ }^{c}\right)$, of the shear force. By this comparative study we aimed to emphasize the way in which $t$ the actual design norms, reflects the observed elements behavior during the experimental tests.


Key words: reinforced concrete, experimental tests, shear force.

## 1. Introduction

Constructions, being goods with the longest term of use, are required to meet throughout their lifetime technical requirements related to: strength, stability, fire safety, exploitation security, durability etc.
Among the decisive factors that lead to the fulfillment of these technical requirements, it is possible to recall those related to the choice of the materials from which the building elements are made, namely their mechanical characteristics, the design mode, the technology of construction and, last but not least, the exploitation way and conditions.
Reinforced concrete, due to its qualities, such as: the ability to achieve any form, good durability under normal operating conditions, fire resistance, good behavior on loads, is the most widely used material in building construction. However, there are two disadvantages such as high thermal and acoustic conductivity as well as high weight. But

[^0]these drawbacks can be removed by using lightweight concrete (concrete made with lightweight aggregates) due to its advantages over conventional one, namely: low weight, improved thermal insulation, better fire behavior and on seismic action etc. [1].

The beneficial use of lightweight concrete in building structural elements requires a thorough knowledge of its behavior on various actions, in particular the behavior at the limit state design in inclined sections as well as elements behavior at the shear force.
In this context, in a comparative analysis of the load-bearing capacity experimentally determined $\left(V_{I R d}{ }^{e}\right)$ and by calculus $\left(V_{I R d}{ }^{c}\right)$, based on the $V_{I R d}{ }^{e} / V_{I R d}{ }^{c}$ ratio, there is emphasized the fidelity with which the current design rules reflects the actual behavior of reinforced lightweight concrete at shear force [2].

## 2. Hypotheses, Principles and Calculation

### 2.1. Mechanical characteristics used for element calculation

Concrete compressive strength is defined by its resistance class, which represents the characteristic resistance (with $5 \%$ fraction) on the cylinder $\left(f_{c k}\right)$ or on the cubes ( $f_{c k, ~ c u b e)}$ ) determined at 28 days. For lightweight concrete, the resistance classes are preceded by the LC symbol, and the lightweight concrete strength characteristics are additionally marked with the / symbol. Some of them have distinct values and others are obtained by multiplying those of ordinary concrete with a coefficient $\eta_{1}$ which depends on the dry density $\rho$ of the lightweight concrete corresponding to the upper limit [2, 4-6]:

$$
\begin{equation*}
\eta_{1}=0.40+0.60 \cdot \rho \cdot 2200^{-1} \tag{1}
\end{equation*}
$$

Strengths values to tension and compression are according to relations:

$$
\begin{align*}
& \mathrm{f}_{\mathrm{lcd}}=\alpha_{l c c} \cdot f_{l c k} \cdot\left(\gamma_{C}\right)^{-1}  \tag{2}\\
& \mathrm{f}_{\text {lctd }}=\alpha_{l c t} \cdot f_{l c t k} \cdot\left(\gamma_{c}\right)^{-1} \tag{3}
\end{align*}
$$

Where $\alpha_{l c c}, \alpha_{l c t}$ are coefficients with recommended value $0.85, f_{l c k}$ is the characteristic compressive cylinder strength of lightweight concrete, $f_{l c t k}$ is the characteristic axial tensile strength of concrete and $\gamma_{c}$ is a partial safety factor for concrete, equal to 1.5 for permanent and transitory design situations and 1.2 for accidental loads [4-6].

### 2.2. Calculation principles for reinforced lightweight concrete elements in inclined sections, at shear force

In order to determine stresses under the effect of the shear force, the static theorem of plasticity theory is used, according to which the real structural element is replaced by a simplified model, fictitious, that ensures the balance between external loads and stresses, satisfying furthermore the plasticizing conditions.

The model can be an isostatic truss made up of compressed concrete bars and tensed steel bars, as shown in Figure 1 [2, 3-4].


A - compression chord, $B$ - struts, $C$ - tensile chord, $D$ - shear reinforcement


Fig. 1. Truss model for calculation
In Figure $1 \alpha$ is the angle between shear reinforcement and the tensile chord, $\vartheta$ is the angle between the compressed cross stud and the tensile chord with a value limited by the relation $1 \leq \cot \vartheta \leq 2.5, F_{t d}$ is the design value of the tensile force from the longitudinal reinforcement, $F_{c d}$ is the design value of the compression force from the concrete in the direction of the longitudinal axis of the element, $b_{w}$ is the minimum width of the cross-section, $d$ is the effective height of the cross-section and $z$ is lever arm of internal forces; its value can be approximated to $0,9 \cdot d[2,4]$.

### 2.3. Calculation relations for reinforced lightweight concrete elements in inclined sections, at shear force [2, 4-6]

The main formula for shear force calculation is:

$$
\begin{equation*}
V_{E d} \leq V_{I R d} \tag{4}
\end{equation*}
$$

Where $V_{E d}$ is the design value of shear force, $V_{I R d}$ is the load-bearing capacity at shear force in inclined sections.

The load-bearing capacity at shear force of a section without transverse reinforcement $V_{I R d, c}$ is according to Equation (6):

$$
\begin{align*}
& V_{I R d}^{c}=V_{I R d, c}+V_{R d, s}  \tag{5}\\
& V_{I R d, c}=\left[c_{l R d, c} \cdot \eta_{1} \cdot k \cdot\left(100 \cdot \rho_{1} \cdot f_{l c k}\right)^{1 / 3} \cdot 2 \cdot d \cdot\left(a_{v}\right)^{-1}\right] \cdot b_{w} \cdot d \leq 0.5 \cdot \eta_{1} \cdot v_{l} \cdot f_{l c d} \cdot b_{w} \cdot d  \tag{6}\\
& C_{I R d}=0.15 \cdot\left(\gamma_{c}\right)^{-1}  \tag{7}\\
& k=1+\left(200 \cdot d^{-1}\right)^{1 / 2}  \tag{8}\\
& v_{l}=0.5 \cdot\left(1-f_{l c k} \cdot 250^{-1}\right) \tag{9}
\end{align*}
$$

where $\gamma_{c}$ is a partial factor for concrete equal to $1.5, d$ is measured in millimeter, $\rho_{1}$ is the percentage of longitudinal reinforcement, $\eta_{1}, f_{l c k}$ and $f_{l c d}$ have been define in paragraph 2.1, $a_{v}$ is the distance between bearings or face of support and face of load, $b_{w}$ and $d$ are according to figure $1, V_{R d, s}$ is the shear resistance of a member governed by the yielding of shear reinforcement and $V_{I R d, s}$ corresponding to the lightweight concrete is given in equation (10) [2, 4]:

$$
\begin{equation*}
V_{l R d, s}=\frac{A_{s w}}{S}\left(z \cdot f_{y w d} \cdot \cot \theta\right) ; 1 \leq \cot \theta \leq 2.5 \tag{10}
\end{equation*}
$$

where $A_{s w}$ is the area of punching shear reinforcement in one perimeter around the column, $s$ is the stirrup spacing and $f_{y w d}$ is the design yield strength of the shear reinforcement.
The physical-mechanical characteristics considered in the calculation relations for the determination of the load bearing capacity $\left(V_{I R d}{ }^{c}\right)$ at shear force are the results obtained when testing samples (at the same age as the beams at the time of the test).

## 3. Validation by Testing Procedure

In order to highlight the behavior of the lightweight concrete elements in the predominant shear force load with reference in particular to the load-bearing capacity in inclined sections, experimental tests were carried out for a number of 16 lightweight reinforced concrete beams having a cross section of $15 \times 30 \mathrm{~cm}$ and the opening of 3.00 m . The total length of the beams was 3.40 m , to ensure a sufficient extension of the reinforcements over the beams' supports.
The longitudinal and transverse reinforcement of the beams was made of PC 52 steel in the form of welded fabrics. Two welded fabrics of the same type were used for each beam, having cross bars (cross reinforcement) of the same diameter ( $\varnothing 6 \mathrm{~mm}$ for all elements) and at the same distance along the length of a wire fabric mesh.
At the ends of the beams, additional measures were taken to ensure anchoring of the reinforcement in concrete (welding of PC 52 steel $\varnothing 52 \mathrm{~mm}$ steel coupons between reinforcement carcasses).

From place to place (at about 80 cm ), the reinforcement frameworks were interlocked by means of transversal joists $\emptyset 4 \ldots 5 \mathrm{~mm}$, arranged horizontally.

The experimental elements were made under laboratory conditions, using metal formwork, and the preparation and compacting of the concrete by mechanical means.

The static load application scheme was that of a simply supported beam, loaded with two symmetrically arranged concentrated forces of equal size, aiming to achieve beams shear apertures $\left(a_{v} / d\right)$ of $1.0,1.5$ and 2.5 . The beams were tested by applying $1 / 10$ increments of the assumed dead load.
When making the elements, concrete samples were taken to determine their physical and mechanical characteristics, namely: 3 cubes having the 14.1 cm side, 3 prisms $10 \times 10 \times 55 \mathrm{~cm}$ and 3 cylinders with the base area $200 \mathrm{~cm}^{2}$ and a height of 32 cm .
The results obtained by testing the samples (at the same age as the beams at the time of the test) represent the physical and mechanical characteristics considered in the calculation relations for the determination of the load bearing capacity $\left(V_{I R d}{ }^{c}\right)$.
Table 1 presents a comparative analysis of the experimental and calculation values of the load-bearing capacity at shear force $\left(V_{I R d}{ }^{e} / V_{I R d}{ }^{c}\right)$ for lightweight concrete elements which highlights how design rules accurately reflect the actual behavior of elements in different states limit under the action of the shear force [2, 4-6].

Comparative analysis and calculation values
Table 1

|  | Code | Concrete type | $V_{\text {IRd }}{ }^{e}$ <br> $[\mathrm{~N}]$ | $V_{\text {IRd }}{ }^{c}$ <br> $[\mathrm{~N}]$ | $V_{\text {IRd }}{ }^{e} / V_{\text {IRd }}{ }^{c}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | G1-1 | lightweight | 150000 | 71612 | 2.04 |
| $\mathbf{2}$ | G1-2 | lightweight | 120000 | 79963 | 1.50 |
| $\mathbf{3}$ | G1-3 | lightweight | 150000 | 78451 | 1.91 |
| $\mathbf{4}$ | G1-4 | lightweight | 165000 | 88260 | 1.86 |
| $\mathbf{5}$ | G1-5 | lightweight | 195000 | 132297 | 1.47 |
| $\mathbf{6}$ | G2-1 | lightweight | 135000 | 82012 | 1.64 |
| $\mathbf{7}$ | G2-2 | lightweight | 150000 | 98687 | 1.52 |
| $\mathbf{8}$ | G2-3 | lightweight | 150000 | 178638 | 0.84 |
| $\mathbf{9}$ | G2-4 | lightweight | 150000 | 86872 | 1.72 |
| $\mathbf{1 0}$ | G2-5 | lightweight | 150000 | 99528 | 1.50 |
| $\mathbf{1 1}$ | G3-1 | lightweight | 90000 | 98009 | 0.92 |
| $\mathbf{1 2}$ | G3-2 | lightweight | 90000 | 121361 | 0.74 |
| $\mathbf{1 3}$ | G3-3 | lightweight | 100000 | 99743 | 1.00 |
| $\mathbf{1 4}$ | G3-4 | lightweight | 100000 | 123413 | 0.81 |
| $\mathbf{1 5}$ | G3-5 | lightweight | 120000 | 99474 | 1.20 |
| $\mathbf{1 6}$ | G3-6 | lightweight | 110000 | 122398 | 0.89 |
| Average |  |  |  |  |  |

## 4. Conclusions

The comparative analysis of the load bearing capacity at shear force experimentally determined ( $V_{I R d}{ }^{e}$ ) and by calculation ( $V_{I R d}{ }^{c}$ ), based on the ( $V_{I R d}{ }^{e} / V_{I R d}{ }^{c}$ ) ratio presented in

Table 1 highlights how design rules reflect in an accurate way the actual behavior of elements in different states limit under the action of the shear force, leading to the following conclusions.
The load-bearing capacity determined experimentally by breaking the beams is, for the vast majority of the tested elements, higher than calculated one for limit state and with the design resistances of the materials determined by test specimens.
The $V_{I R d}{ }^{e} / V_{I R d}{ }^{c}$ ratio varies between 0.74...2.04 (average 1.34) and represent the partial safety factor related to the materials properties.
If we also take into account the partial factor related to the loads with an average value of 1.4 , we reach a global safety level of 1.8 , which can be considered satisfactory, being generally used as a global safety factor in design.

## References

1. Mizgan, P., Onet, T.: Beton ușor (Lightweight concrete) Ed. Universității Transilvania din Brașov, 2005.
2. Mizgan, P.: Comportarea şi calculul elementelor din beton uşor la forţă tăietoare (Behavior and calculation of the lightweight concrete elements at shear force) PhD Thesys, 2006.
3. Mizgan, P., Onet, T.: Comportarea şi calculul elementelor din beton uşor armat în secţiuni înclinate după prevederile sr en 1992-1 (Behaviour and shear calculation of light reinforced elements according to sr en 1992-1 standard) International scientific conference CIBv 2010, Brasov.
4. SR EN 1992-1-1 Proiectarea structurilor de beton (Design of concrete structures) 2006
5. SR EN 1992-1-1/AC Proiectarea structurilor de beton - Erată (Design of concrete structures - Erratum) 2008
6. SR EN 1992-1-1-NB Proiectarea structurilor de beton (Design of concrete structures) 2008.

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