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SHORT PLANE BEARINGS LUBRICATION APPLIED ON SILENT CHAIN JOINTS

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Abstract: *This paper describes the theoretical pressure distribution and loading capacity of the silent chain joints. The lubricating contact between pin and toothed cleat (silent chains) is modelled with Ocvirk's short bearing solution which is based on the assumption that the contact surface between the two elements on width direction is significantly smaller than on length direction. This paper is presenting the steps to obtain the final formula for pressure distribution, numerical calculations are applied on the case of a silent chain. The loading capacity is computed depending on lubricant thickness and rotational speed in order to predict the type of lubrication that is present between the pin and tooth cleats.*

Key words: *chain joint, lubrication, Ocvirk's pressure distribution.*

1. Introduction

Silent chains are mostly used as timing chains on internal combustion engines. They are transmitting rotational movement and torque from the crank shaft to the camshaft, oil pump, and high pressure pump, ensuring the correct opening and closing of the intake and exhaust valves. Friction loss in these components is of maximal importance [1], [2].

The components of the most common silent chain are presented in Figure 1 [3]. The inner cleats 1, are mounted with clearance on the pin 4, ensuring the relative movement between them. The outer cleats 2 and the middle cleat 3 are mounted with interference fit on the pin 4, helping to guide the chain on the sprocket.

The pin - inner cleat joint is a typical case of oscillating plane bearing [4]. The research in the direction of diminishing friction and wear of the pin - inner cleat joint is very important and must start with determining the type of lubrication of the joint.

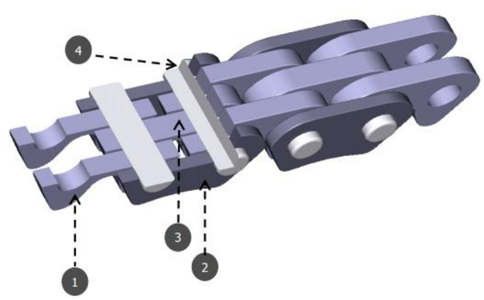

Fig. 1. *Silent chain [3]*

The joint works as a plane bearing in the phases of entering and getting out of each sprocket, with an oscillating rotation. The amplitude of rotation is equal with the pitch angle of each sprocket.

The lubrication theory [4-6] dealing with plane bearings is so vast, with several

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models based on different assumptions and simplifications, but all theoretical models of lubrication, describing the process of hydrodynamic pressure generation can be described mathematically with the help of the Reynolds equation.

The Reynolds equation is a partial differential equation describing the pressure distribution of thin viscous fluid film bearings, within bodies that are fully separated. The general equation is:

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)= \\
&=\frac{\partial}{\partial x}\left(\frac{\rho h(u_a+u_b)}{2}\right)+ \\
&+\frac{\partial}{\partial x}\left(\frac{\rho h(\upsilon_a+\upsilon_b)}{2}\right)+\rho(w_a-w_b)- \\
&-\rho u_a\frac{\partial h}{\partial x}-\rho\upsilon_a\frac{\partial h}{\partial y}+h\frac{\partial\rho}{\partial z}.
\end{aligned} \tag{1}$$

The parameters from (1) are: p – the fluid film pressure; x and y are the coordinates of the two axes describing bearing length and width; z fluid film thickness coordinate; h fluid film thickness; η is the fluid viscosity; ρ is the fluid density; u, υ and w are the bounding body velocities in x, y, z directions; a and b the top and bottom boundaries. The assumptions considered in Reynolds equation is: the fluid is Newtonian; the viscous forces dominate over the fluid inertia forces; the fluid body forces are negligible; the variation of pressure across the fluid film is small; the fluid film thickness is much less than the length and width.

The aim of this present paper is to study the theoretical pressure distribution and loading capacity of the joints of a silent chain. The oscillating plane bearing is a dynamic, very complex case. The approach of this paper is looking on a simplified static model that doesn't leave the pressure to develop on tangential direction. The resulted load capacity will be diminished and could be compared with the moment when the lubricating film has the maximum height on the oscillating movement.

2. Applied Ocvirk's short bearing solution

2.1. Theoretical model

Ocvirk's short bearing solution [7] is a simplified model of Reynolds equation. This model is usable for bearings with dimensions relatively reduced in width where the lubricant loss has a big role. In this case the pressure peak is obtained on a reduced longitudinal section of the bearing, due to the incapacity to maintain the lubricant between the friction surfaces. The Ocvirk's short bearing solution is based on the assumption that the contact surface on x direction is so reduced compared to the y direction that it can be neglected; the lubricant thickness tends to 0, it can be also neglected.

Figure 2 presents the diagram of a general plane bearing lubrication.

The pressure distribution is varying within an angle φ from 0 to 180 degrees.

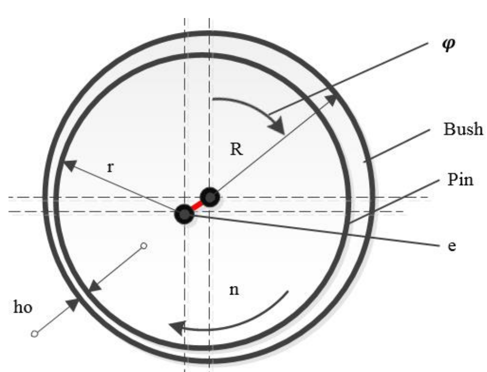

Fig. 2. *Cylindrical joint with minimum lubricant thickness*

The theoretical model can be applied to silent chain joints where providing the necessary oil in the pin ï mobile toothed cleats clearance is the main interest for improvement. The contact between the inner cleats and the pin is significantly reduced also compared to the contact surface between the pin and bush of bush chains. This diminishes the capacity to create fluid film in the direction of rotation (x), while lubricant losses at the edges of the cleats are important.

The unidirectional velocity approximation of Reynoldôs equation is:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\cdot\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\cdot\left(\frac{h^3}{12\cdot\eta}\cdot\frac{\partial p}{\partial y}\right)= = u\cdot\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t}; \quad (2)$$

The computation starts with the following mathematical assumptions:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) << \frac{\partial}{\partial y}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial y}\right); \quad (3)$$

$$\frac{\partial h}{\partial t}\to 0. \quad (4)$$

The Reynoldôs unidirectional velocity approximation (1) becomes Ocvrikôs equation:

$$\frac{\partial}{\partial y}\left(\frac{h^3}{12\eta}\cdot\frac{\partial p}{\partial y}\right)=u\frac{\partial h}{\partial x}. \quad (5)$$

Based on the diagram from Figure 1, the following relations can be drawn:

$$u=\frac{\pi n}{30}R; \quad (6)$$

$$x=R\cdot\varphi; \quad (7)$$

$$c=R-r; \quad (8)$$

$$e=R-r-h_0; \quad (9)$$

$$h=c+e\cdot\cos\varphi, \quad (10)$$

where u is the relative average speed between the inner cleats and pin in rotational movement, e is the eccentricity between the pin and the toothed cleats and h_0 is the minimum lubricant thickness.

After integration, (5) becomes:

$$\frac{h^3}{12\eta}\frac{\partial p}{\partial y}=u\frac{\partial h}{R\partial\varphi}+C_1. \quad (11)$$

Applying the boundary condition:

$$\frac{\partial p}{\partial y}=0\text{, at } y=0, \quad (12)$$

it results $C_1 = 0$ and

$$\frac{\partial p}{\partial y}=\frac{12\eta u}{h^3 R}\frac{\partial h}{\partial\varphi}y. \quad (13)$$

After the second integration, the equation of pressure is:

$$p=\frac{12\cdot\eta\cdot u}{h^3\cdot R}\cdot\frac{\partial h}{\partial\varphi}\cdot\frac{y^2}{2}+C_2. \quad (14)$$

By applying the boundary conditions for the pressure, at the limits of width of the cleats, pressure p at $\pm\frac{B}{2}$ is equal to 0, where B is the width of the toothed cleats, the constant C_2 becomes:

$$C_2=-\frac{12\eta u}{h^3 R}\frac{\partial h}{\partial\varphi}\frac{B^2}{8} \quad (15)$$

and (14) becomes:

$$p = \frac{6\eta u}{h^3 R}\frac{\partial h}{\partial \varphi}\left(y^2 - \frac{B^2}{4}\right). \tag{16}$$

Integrating h from (10), the pressure distribution is

$$p = \frac{6\eta u}{R}\frac{e\sin\varphi}{(c + e\cos\varphi)}\left(\frac{B^2}{4} - y^2\right), \tag{17}$$

expressing a function $p(\varphi, y)$ on intervals $\varphi \in [0, 180]$ and $y \in \left[-\frac{B}{2}, +\frac{B}{2}\right]$.

The loading capacity F of the bearing is

$$F = \iint_{\varphi\, y} p\,dy\,d\varphi. \tag{18}$$

The type of lubrication represents also a point of interest. The dimensionless film thickness ratio Λ [6], indicating the type of lubrication is computed as

$$\Lambda = \frac{h_0}{\sqrt{R_{a\,pin}^2 + R_{a\,cleat}^2}} \tag{19}$$

where: R_a is the average surface roughness of the pin and toothed cleats.

The range of values of [6] indicates:

$\Lambda < 1$ - boundary lubrication;

$1 < \Lambda < 5$ - mixed lubrication;

$3 < \Lambda < 10$ - elasto-hydrodynamic lubrication (EHL);

$5 < \Lambda < 100$ - hydrodynamic lubrication(HDL).

2.1. Results – Pressure Distribution

The above presented mathematical model for bearing lubrication has been applied for the case of a silent chain joint, using as input parameters three values for minimum lubricant thickness h_0 (0.1 μm, 0.2 μm, 0.3 μm); and three values of the rotational speed of the chain sprocket n (1000 rpm, 3000 rpm, 5000 rpm). Table 1 presents the results for the case of minimum loading: h_0 = 0.3 μm (maximum from the three values) and rotational speed n = 1000 rpm (minimum of the three values). For the same case, Figure 3 shows the pressure distribution and Figure 4 shows the peak pressure at the middle of the inner cleat.

Table 1

h_0 [m]	3.00E-07
n [rpm]	1000
B [m]	0.0002
R [m]	0.00187
r [m]	0.00182
c [m]	0.00005
η [Pas]	0.06
ω [rad/s]	104.7197551
u [m/s]	0.190066
e [m]	4.97E-05
R_a [m]	8.00E-07
Λ	3.7500
F [N]	0.223

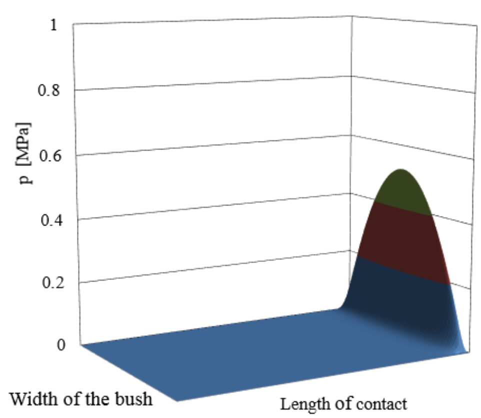

Fig. 3. *Pressure distribution for n =1000 rpm, h_0 = 0.3μm*

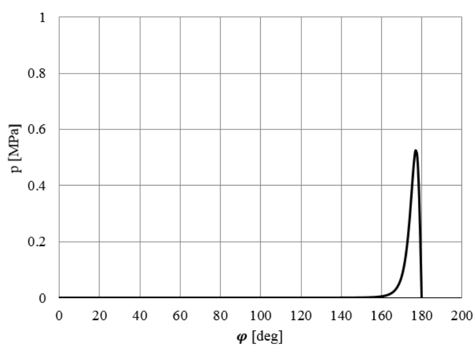

Fig. 4. *Peak pressure at the middle of the inner cleat for n = 1000 rpm at h_0 = 0.3µm*

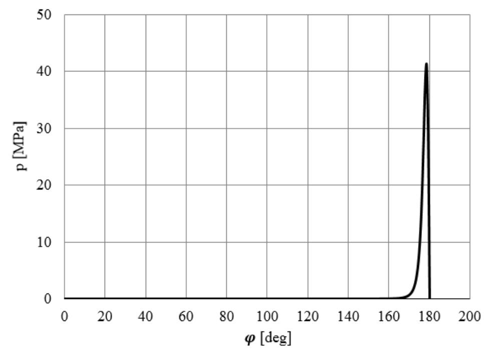

Fig. 6. *Peak pressure at the middle of the inner cleat for n = 5000 rpm, h_0 = 0.1 µm*

Table 2

h_0 [m]	1E-07
n [rpm]	5000
B [m]	0.0002
R [m]	0.00187
r [m]	0.00182
c [m]	0.00005
η [Pas]	0.06
ω [rad/s]	523.599
u [m/s]	0.9503
e [m]	4.99E-05
R_a [m]	8.00E-08
Λ	1.250
F [N]	17.364

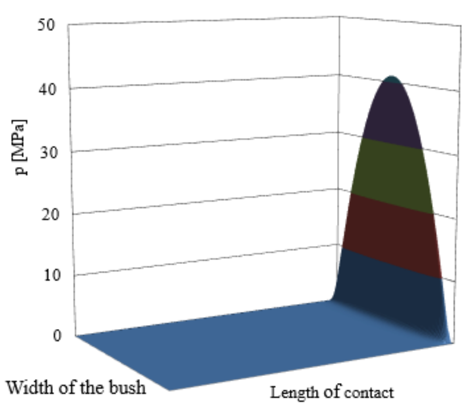

Fig. 5. *Pressure distribution for n = 5000 rpm at h_0 = 0.1 µm*

2.1. Results – Type of lubrication

The analysis of the type of lubrication must involve the dependence between minimum lubricant thickness h_0, loading capacity F and rotational speed n. Figure 7 shows the loading capacity depending on rotational speed, for three values of minimum lubricant thickness.

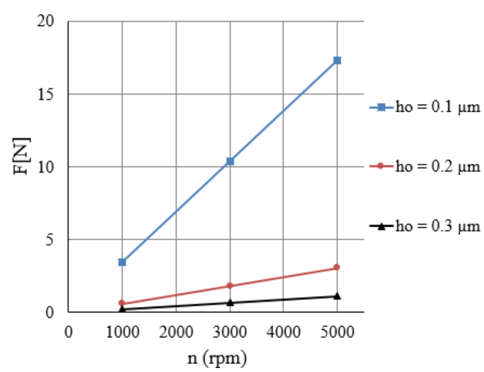

Fig. 7. *Loading capacity depending on rotational speed for constant minimum lubricant thickness.*

Figure 8 presents the computed dimensionless film thickness ratio for average surface roughness of pin and toothed mobile cleat R_a = 0.08 µm depending on load, for three values of rotational speed.

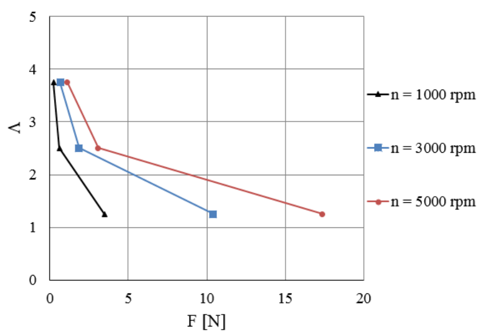

Fig.8. *Film thickness ratio depending on rotational speed for constant rotational speed.*

The results of the theoretic analysis for the pin-inner cleat contact parameters determination, based on the lubrication theory show that:

1) The maximum pressure values are obtained at the areas very near to the minimum lubricant thickness;

2) Looking at the values of the maximum pressure, limited to 42 MPa, we can conclude that the theory of hydrodynamic lubrication is applied correctly; there is no need to consider elasto-hydrodynamic lubrication;

3) The normal condition of working for such joints is in the range [8]: rotational speed n from 500 to 5000 rpm, normal force F from 50 to 500 N; The small values of the lubricant film thickness, for very small load capacity, indicate that it is nearly impossible to obtain conditions for hydrodynamic lubrication. The results show clearly boundary or mixed lubrication.

3. Conclusion

The main conclusion from theoretical analyse of the lubrication conditions of chain joints is that hydrodynamic lubrication is almost impossible to achieve.

However, this conclusion must be proved by experimental testing.

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